Acceleration and Velocity Sensing from Measured Strain

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What the technology does

Problem Statement

- Improving fuel efficiency for an aircraft
  - Reducing **weight** or **drag**
    - Similar effect on fuel savings
  - Multidisciplinary design optimization (design phase) or active control (during flight)

- Real-time measurement of deflection, slope, and loads in flight are a valuable tool.
  - Active flexible motion control
  - Active induced drag control

- Wing deflection and slope (complete degrees of freedom) are essential quantities for load computations during flight.
  - Loads can be computed from the following governing equations of motion.

$$[\mathbf{M}]\{\ddot{\boldsymbol{q}}(\boldsymbol{t})\} + [\mathbf{G}]\{\dot{\boldsymbol{q}}(\boldsymbol{t})\} + [\mathbf{K}]\{\boldsymbol{q}(\boldsymbol{t})\} = \{\boldsymbol{Q}_{\boldsymbol{a}}(\boldsymbol{Mach}, \boldsymbol{q}(\boldsymbol{t}))\}$$

    - Internal Loads: using finite element structure model
      - ✓ $[\mathbf{M}]\{\ddot{\boldsymbol{q}}(\boldsymbol{t})\}$, $[\mathbf{G}]\{\dot{\boldsymbol{q}}(\boldsymbol{t})\}$, $[\mathbf{K}]\{\boldsymbol{q}(\boldsymbol{t})\}$: Inertia, damping, and elastic loads
    - External Load: using unsteady aerodynamic model
      - ✓ $\{\boldsymbol{Q}_{\boldsymbol{a}}(\boldsymbol{Mach}, \boldsymbol{q}(\boldsymbol{t}))\}$: Aerodynamic load

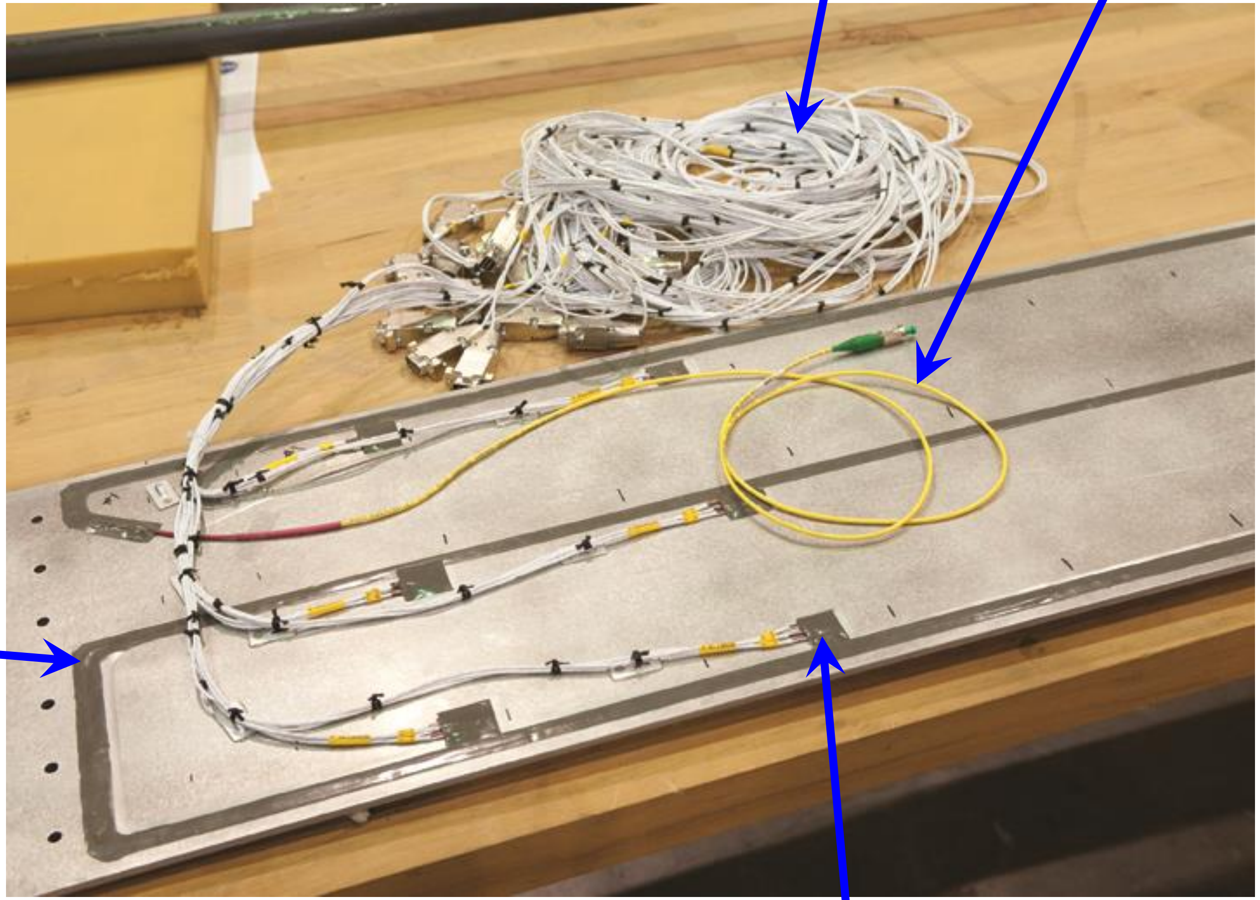
- Traditionally, strain over the wing are measured using strain gages.
  - Cabling would create **weight and space limitation** issues.
  - A **new innovation** is needed. **Fiber optic strain sensor** (FOSS) is an ideal choice for **aerospace** applications.

$$\boldsymbol{q}(\boldsymbol{t}) = \begin{Bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix}$$

$\delta_x, \delta_y, \delta_z$ → Deflection

$\theta_x, \theta_y, \theta_z$ → Slope (angle)

Complete degrees of freedom



Wing deflection & slope at time t will be computed from measured strain.

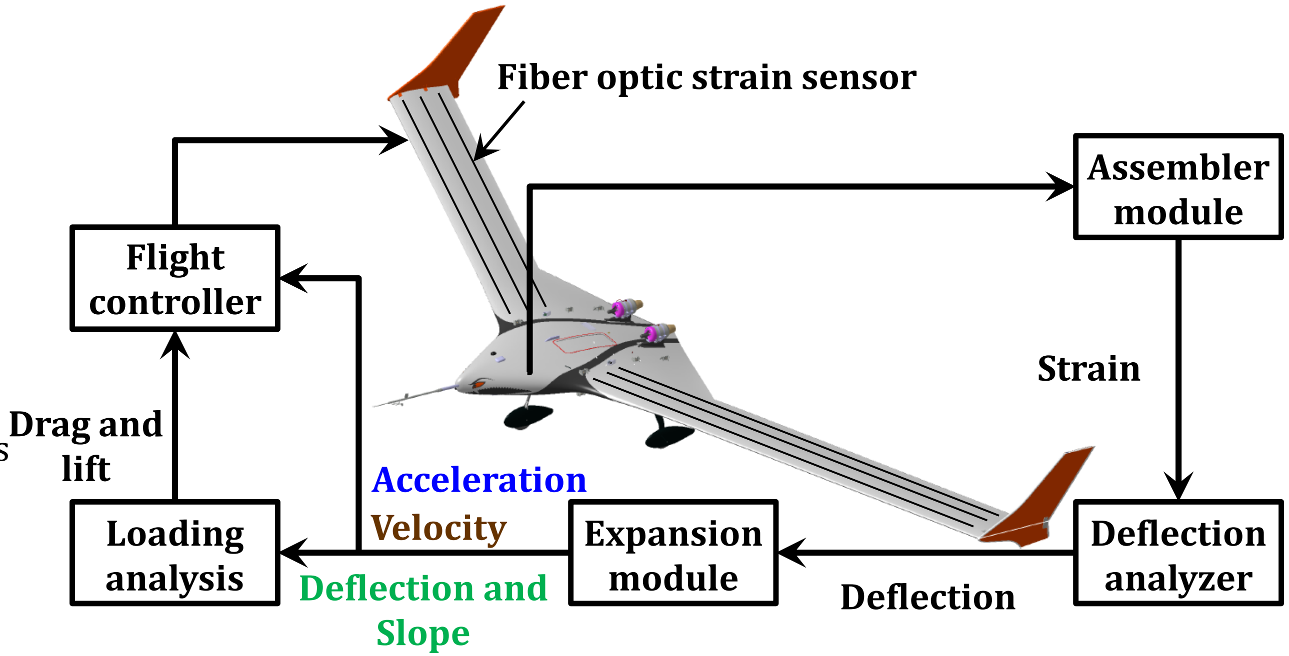
Previous technologies

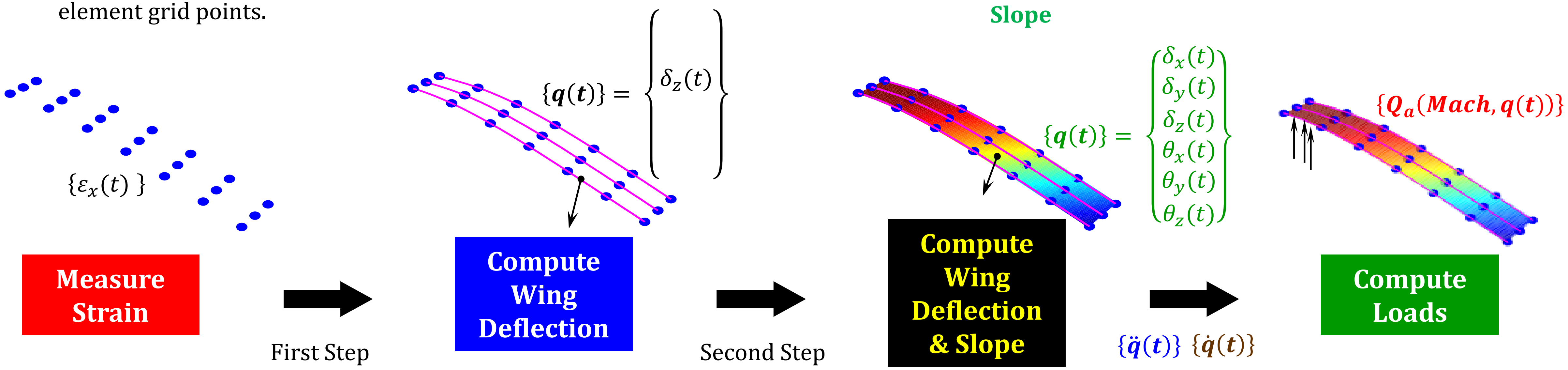
- Liu, T., Barrows, D. A., Burner, A. W., and Rhew, R. D., "Determining Aerodynamic Loads Based on Optical Deformation Measurements," *AIAA Journal,* Vol.40, No.6, June 2002, pp.1105-1112
  - **NASA LRC;** Application is limited for **"beam"; static deflection & aerodynamic loads**
- Shkarayev, S., Krashantisa, R., and **Tessler**, A., "An Inverse Interpolation Method Utilizing In-Flight Strain Measurements for Determining Loads and Structural Response of Aerospace Vehicles," Proceedings of Third International Workshop on Structural Health Monitoring, **2001**
  - **University of Arizona** and **NASA LRC;** "**Full 3D"** application; **strain matching optimization; static deflection & loads**
- Kang, L.-H., Kim, D.-K., and Han, J.-H., "Estimation of Dynamic Structural Displacements using fiber Bragg grating strain sensors," **2007**
  - **KAIST;** displacement-strain-transformation (DST) matrix; Use **strain mode shape**; Application was based on **beam structure**; **dynamic deflection**
- Igawa, H. et al., "Measurement of Distributed Strain and Load Identification Using 1500 mm Gauge Length FBG and Optical Frequency Domain Reflectometry," 20th International Conference on Optical Fibre Sensors, **2009**
  - **JAXA; using inverse analysis. "Beam" application only; static deflection & loads**
- Ko, W. and Richards, L., "Method for real-time structure shape-sensing," US Patent #7520176B1, April 21, **2009**
  - **NASA AFRC; closed-form equations** (based on **beam theory**); **static deflection**
- Richards, L. and Ko, W. , "Process for using surface strain measurements to obtain operational loads for complex structures," US Patent #7715994, May 11, **2010**
  - **NASA AFRC; "sectional" bending moment, torsional moment, and shear force** along the **"beam"**.
- Moore, J.P., "Method and Apparatus for Shape and End Position Determination using an Optical Fiber," U.S. Patent No. 7813599, issued October 12, **2010**
  - **NASA LRC; curve-fitting**; **static deflection**
- Park, Y.-L. et al., "Real-Time Estimation of Three-Dimensional Needle Shape and Deflection for MRI-Guided Interventions," *IEEE/ASME Transactions on Mechatronics*, Vol. 15, No. 6, **2010**, pp. 906-915
  - **Harvard University**, **Stanford University**, and **Howard Hughes Medical Institute**; Uses **beam theory; static deflection & loads**
- Carpenter, T.J. and Albertani, R., "Aerodynamic Load Estimation from Virtual Strain Sensors for a Pliant **Membrane Wing**," *AIAA Journal,* Vol.53, No.8, August 2015, pp.2069-2079
  - **Oregon State University; Aerodynamic loads** are estimated from measured strain using virtual strain sensor technique.
- Pak, C.-g., "Wing Shape Sensing from Measured Strain," AIAA 2015-1427, AIAA Infotech @ Aerospace, Kissimmee, Florida, January 5-9, **2015**; accepted for publication on *AIAA Journal* (June 29, **2015**); U.S. Patent Pending: Patent App No. 14/482784
  - **NASA AFRC; "Full 3D"** application; based on **System Equivalent Reduction Expansion Process; static deflection**

Technical features of two-step approach: Deflection Computation

Proposed solutions:

- The method for obtaining the deflection over a flexible full 3D aircraft structure was based on the following two steps.
  - First Step: Compute wing deflection along fibers using measure strain data
    - Wing deflection will be computed along the fiber optic sensor line.
    - Strains at selected locations will be "**fitted**".
    - These fitted strain will be integrated twice to have deflection information. (Relative deflection w.r.t. the reference point)
    - This is **a finite element model independent** method.
  - Second Step: Compute wing slope and deflection of entire structures
    - Slope computation will be based on **a finite element model dependent** technique.
    - Wing deflection and slope will be computed at all the finite element grid points.





Technical features of two-step approach : Deflection Computation (continued)

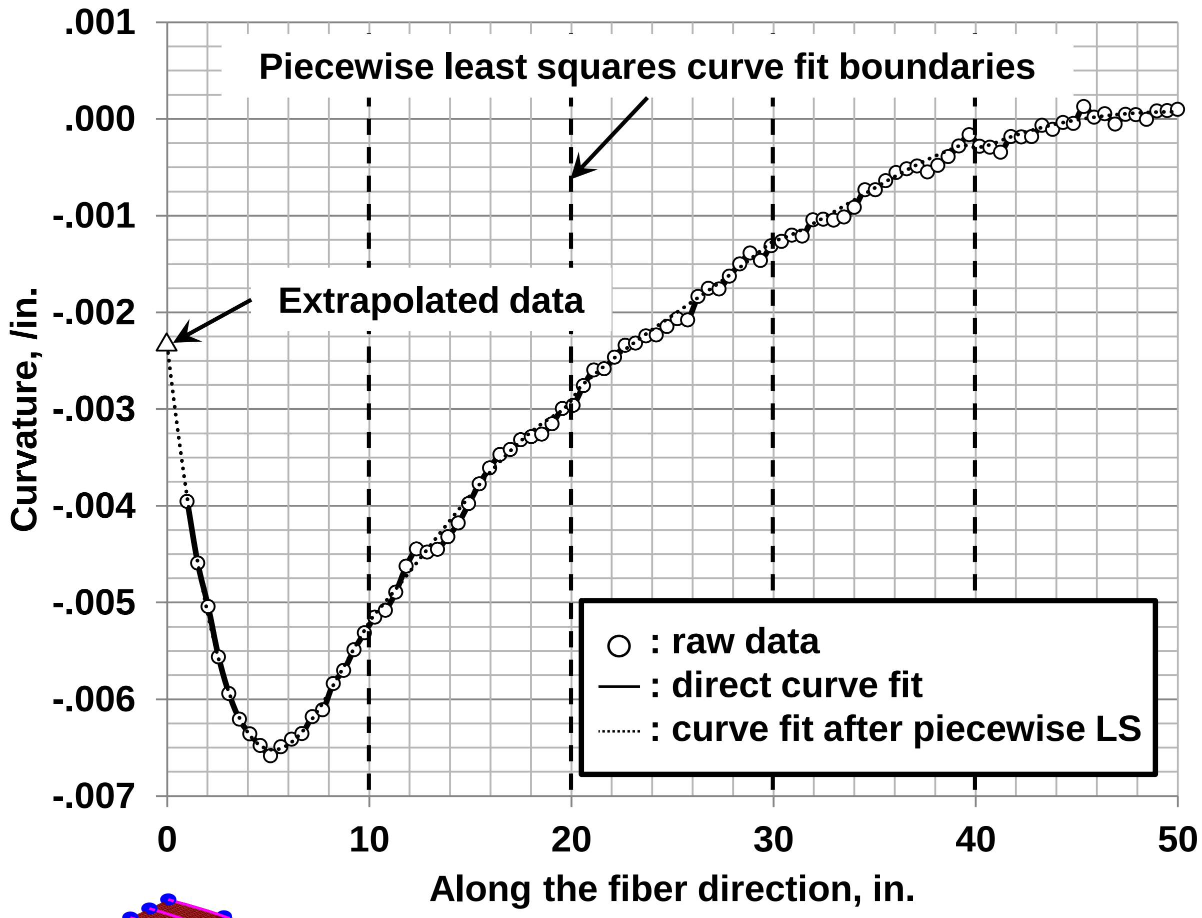
- First Step
  - Use piecewise least-squares method to minimize noise in the measured strain data (strain/offset)
  - Obtain cubic spline (Akima spline) function using re-generated strain data points (assume small motion):

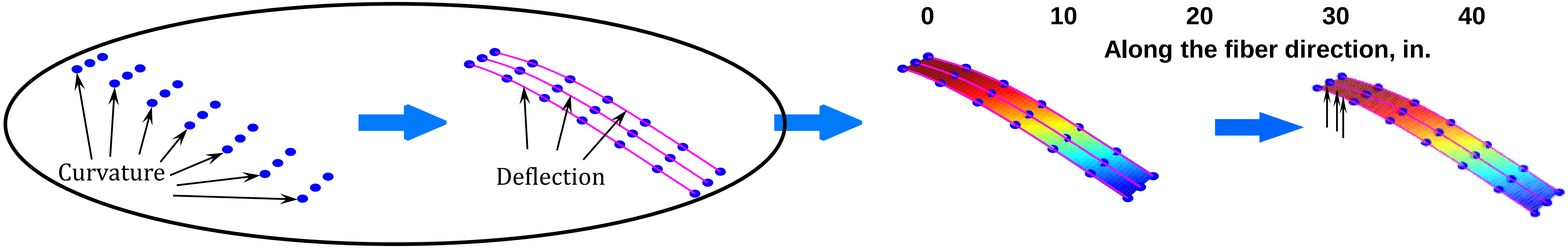
$$\frac{d^2\delta_k}{ds^2} = -\epsilon_k(s)/c(s)$$

  - Integrate fitted spline function to get slope data:

$$\frac{d\delta_k}{ds} = \theta_k\,(s)$$

  - Obtain cubic spline (Akima spline) function using computed slope data
  - Integrate fitted spline function to get deflection data: $\delta_k(s)$





A measured strain is fitted using a piecewise least-squares curve fitting method together with the cubic spline technique.

Technical features of two-step approach : Deflection Computation (continued)

- Second Step: Based on General Transformation
  - Definition of the generalized coordinates vector $\{\boldsymbol{q}\}_k$ and the othonormalized coordinates vector $\{\boldsymbol{\eta}\}_k$ at discrete time k

$$\{\boldsymbol{q}\}_k = \begin{Bmatrix} \boldsymbol{q_M} \\ \boldsymbol{q_S} \end{Bmatrix}_k = [\boldsymbol{\Phi}]\{\boldsymbol{\eta}\}_k = \begin{bmatrix} \boldsymbol{\Phi_M} \\ \boldsymbol{\Phi_S} \end{bmatrix} \{\boldsymbol{\eta}\}_k$$

  - For all model reduction/expansion techniques, there is a relationship between the **master (measured or tested)** degrees of freedom and the **slave (deleted or omitted)** degrees of freedom which can be written in general terms as

$$\{\boldsymbol{q_M}\}_k = [\boldsymbol{\Phi_M}]\{\boldsymbol{\eta}\}_k$$

$$\{\boldsymbol{q_S}\}_k = [\boldsymbol{\Phi_S}]\{\boldsymbol{\eta}\}_k$$

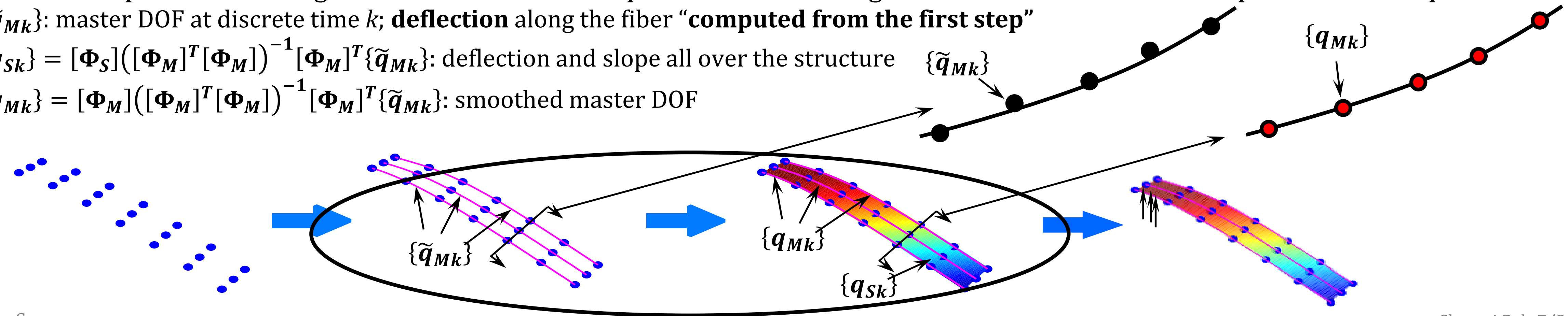
  - Changing master DOF at discrete time k $\{\boldsymbol{q_M}\}_k$ to the corresponding measured values $\{\boldsymbol{\tilde{q}_M}\}_k$

$$\{\boldsymbol{\tilde{q}_M}\}_k = [\boldsymbol{\Phi_M}]\{\boldsymbol{\eta}\}_k$$

$$[\boldsymbol{\Phi_M}]^T\{\boldsymbol{\tilde{q}_M}\}_k = [\boldsymbol{\Phi_M}]^T[\boldsymbol{\Phi_M}]\{\boldsymbol{\eta}\}_k$$

$$\{\boldsymbol{\eta}\}_k = \left([\boldsymbol{\Phi_M}]^T[\boldsymbol{\Phi_M}]\right)^{-1}[\boldsymbol{\Phi_M}]^T\{\boldsymbol{\tilde{q}_M}\}_k \qquad \{\boldsymbol{q}\}_k = \begin{bmatrix} \boldsymbol{\Phi_M} \\ \boldsymbol{\Phi_S} \end{bmatrix}\left([\boldsymbol{\Phi_M}]^T[\boldsymbol{\Phi_M}]\right)^{-1}[\boldsymbol{\Phi_M}]^T\{\boldsymbol{\tilde{q}_M}\}_k$$

- Expansion of displacement using SEREP: kinds of least-squares surface fitting; most accurate reduction-expansion technique
  - $\{\boldsymbol{\tilde{q}_{Mk}}\}$: master DOF at discrete time k; **deflection** along the fiber "**computed from the first step**"
  - $\{\boldsymbol{q_{Sk}}\} = [\boldsymbol{\Phi_S}]\left([\boldsymbol{\Phi_M}]^T[\boldsymbol{\Phi_M}]\right)^{-1}[\boldsymbol{\Phi_M}]^T\{\boldsymbol{\tilde{q}_{Mk}}\}$: deflection and slope all over the structure
  - $\{\boldsymbol{q_{Mk}}\} = [\boldsymbol{\Phi_M}]\left([\boldsymbol{\Phi_M}]^T[\boldsymbol{\Phi_M}]\right)^{-1}[\boldsymbol{\Phi_M}]^T\{\boldsymbol{\tilde{q}_{Mk}}\}$: smoothed master DOF



Technical features of new technology: Acceleration Computation

- From $\{\boldsymbol{q}\}_k = \begin{Bmatrix} \boldsymbol{q}_M \\ \boldsymbol{q}_S \end{Bmatrix}_k = \begin{bmatrix} \boldsymbol{\Phi}_M \\ \boldsymbol{\Phi}_S \end{bmatrix} \{\boldsymbol{\eta}\}_k$ $\quad \{\ddot{\boldsymbol{q}}\}_k = \begin{Bmatrix} \ddot{\boldsymbol{q}}_M \\ \ddot{\boldsymbol{q}}_S \end{Bmatrix}_k = \begin{bmatrix} \boldsymbol{\Phi}_M \\ \boldsymbol{\Phi}_S \end{bmatrix} \{\ddot{\boldsymbol{\eta}}\}_k$
- Assume **simple harmonic motion** for normalized coordinates. $\ddot{\eta}_i(k) = -\omega_i^2 \eta_i(k) \quad i = 1,2,\dots,n$
- Acceleration at discrete time k can be expressed

$$\{\ddot{\boldsymbol{\eta}}\}_k = \begin{Bmatrix} \ddot{\eta}_1(k) \\ \ddot{\eta}_2(k) \\ \vdots \\ \ddot{\eta}_n(k) \end{Bmatrix} = -\begin{bmatrix} \omega_1^2 & 0 & \dots & 0 \\ 0 & \omega_2^2 & \dots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \dots & \omega_n^2 \end{bmatrix} \begin{Bmatrix} \eta_1(k) \\ \eta_2(k) \\ \vdots \\ \eta_n(k) \end{Bmatrix} = -[\boldsymbol{\omega}_i^2]\{\boldsymbol{\eta}\}_k$$

$$\{\ddot{\boldsymbol{q}}\}_k = -\begin{bmatrix} \boldsymbol{\Phi}_M[\boldsymbol{\omega}_i^2] \\ \boldsymbol{\Phi}_S[\boldsymbol{\omega}_i^2] \end{bmatrix} \{\boldsymbol{\eta}\}_k \quad Eq.(9)$$

$$\{\boldsymbol{\eta}\}_k = \left([\boldsymbol{\Phi}_M]^T[\boldsymbol{\Phi}_M]\right)^{-1}[\boldsymbol{\Phi}_M]^T\{\tilde{\boldsymbol{q}}_M\}_k \quad \text{Eq. (6)}$$

- Substituting Eq. (6) into (9) gives

Computed from unsteady strain distribution at a selected point using an on-line parameter estimation technique together with an AutoRegressive Moving Average (ARMA) model

$$\{\ddot{\boldsymbol{q}}\}_k = -\begin{bmatrix} \boldsymbol{\Phi}_M[\boldsymbol{\omega}_i^2] \\ \boldsymbol{\Phi}_S[\boldsymbol{\omega}_i^2] \end{bmatrix} \left([\boldsymbol{\Phi}_M]^T[\boldsymbol{\Phi}_M]\right)^{-1}[\boldsymbol{\Phi}_M]^T\{\tilde{\boldsymbol{q}}_M\}_k$$

$$\{\boldsymbol{q}\}_k = \begin{bmatrix} \boldsymbol{\Phi}_M \\ \boldsymbol{\Phi}_S \end{bmatrix} \left([\boldsymbol{\Phi}_M]^T[\boldsymbol{\Phi}_M]\right)^{-1}[\boldsymbol{\Phi}_M]^T\{\tilde{\boldsymbol{q}}_M\}_k$$

Master DOF at discrete time k; **deflection** along the fiber "**computed from the first step**"

Basis function for least squares surface fitting: eigen function, comparison function, etc.

Technical features of New Technology: Velocity Computation

- From $\{q\}_k = \begin{Bmatrix} q_M \\ q_S \end{Bmatrix}_k = \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \{\eta\}_k$ $\quad \{\dot{q}\}_k = \begin{Bmatrix} \dot{q}_M \\ \dot{q}_S \end{Bmatrix}_k = \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \{\dot{\eta}\}_k$

$$\{\ddot{q}\}_k = -\begin{bmatrix} \Phi_M[\omega_i^2] \\ \Phi_S[\omega_i^2] \end{bmatrix} \left([\Phi_M]^T[\Phi_M]\right)^{-1}[\Phi_M]^T\{\tilde{q}_M\}_k$$

$$\{q\}_k = \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \left([\Phi_M]^T[\Phi_M]\right)^{-1}[\Phi_M]^T\{\tilde{q}_M\}_k$$

- Consider
  - Backward difference: $\{\dot{\eta}\}_k = \dfrac{\{\eta\}_k - \{\eta\}_{k-1}}{\Delta t}$ has "phase shift" issue
  - Central difference: $\{\dot{\eta}\}_k = \dfrac{\{\eta\}_{k+1} - \{\eta\}_{k-1}}{2\Delta t}$ needs future response at time k

- From linear AR model for the i-th orthonormalized coordinate $\eta_i(k) = a_{1i}\eta_i(k-1) + a_{2i}\eta_i(k-2)$
  - Future prediction $\eta_i(k+1)$ at time k $\quad \eta_i(k+1) = a_{1i}\eta_i(k) + a_{2i}\eta_i(k-1)$
  - Central difference becomes $\quad \dot{\eta}_i(k) = \dfrac{a_{1i}\eta_i(k) + (a_{2i}-1)\eta_i(k-1)}{2\Delta t}$
  - AR coefficients a_{1i} & a_{2i} for the i-th mode are computed from the i-th frequency ω_i which are estimated from the parameter estimation

$$\{\dot{q}\}_k \overset{???}{=} \begin{bmatrix} \Phi_M[j\omega_i] \\ \Phi_S[j\omega_i] \end{bmatrix} \left([\Phi_M]^T[\Phi_M]\right)^{-1}[\Phi_M]^T\{\tilde{q}_M\}_k$$

Computed from estimated frequencies

Master DOF at discrete time k; **deflection** along the fiber "**computed from the first step**"

$$\{\dot{q}\}_k = \begin{Bmatrix} \dot{q}_M \\ \dot{q}_S \end{Bmatrix}_k = \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \{\dot{\eta}\}_k$$

$$\dot{\eta}_i(k) = \frac{a_{1i}\eta_i(k) + (a_{2i}-1)\eta_i(k-1)}{2\Delta t}$$

$$\{\eta\}_k = \left([\Phi_M]^T[\Phi_M]\right)^{-1}[\Phi_M]^T\{\tilde{q}_M\}_k$$

$$\{\dot{\eta}\}_k = \begin{Bmatrix} \dot{\eta}_1(k) \\ \dot{\eta}_2(k) \\ \vdots \\ \dot{\eta}_i(k) \end{Bmatrix}$$

Basis function for least squares surface fitting: eigen function, comparison function, etc.

Computational Validation

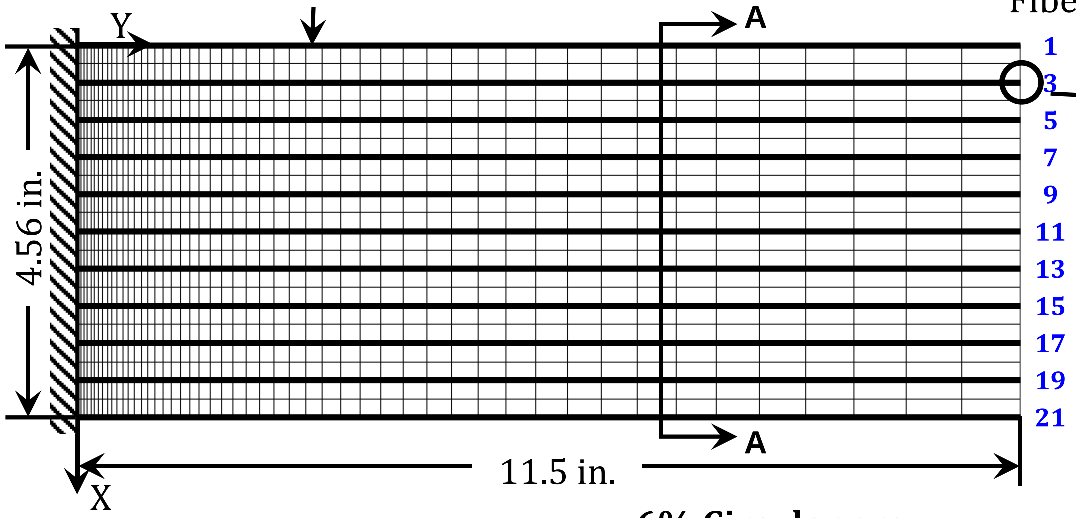
Cantilevered rectangular wing model

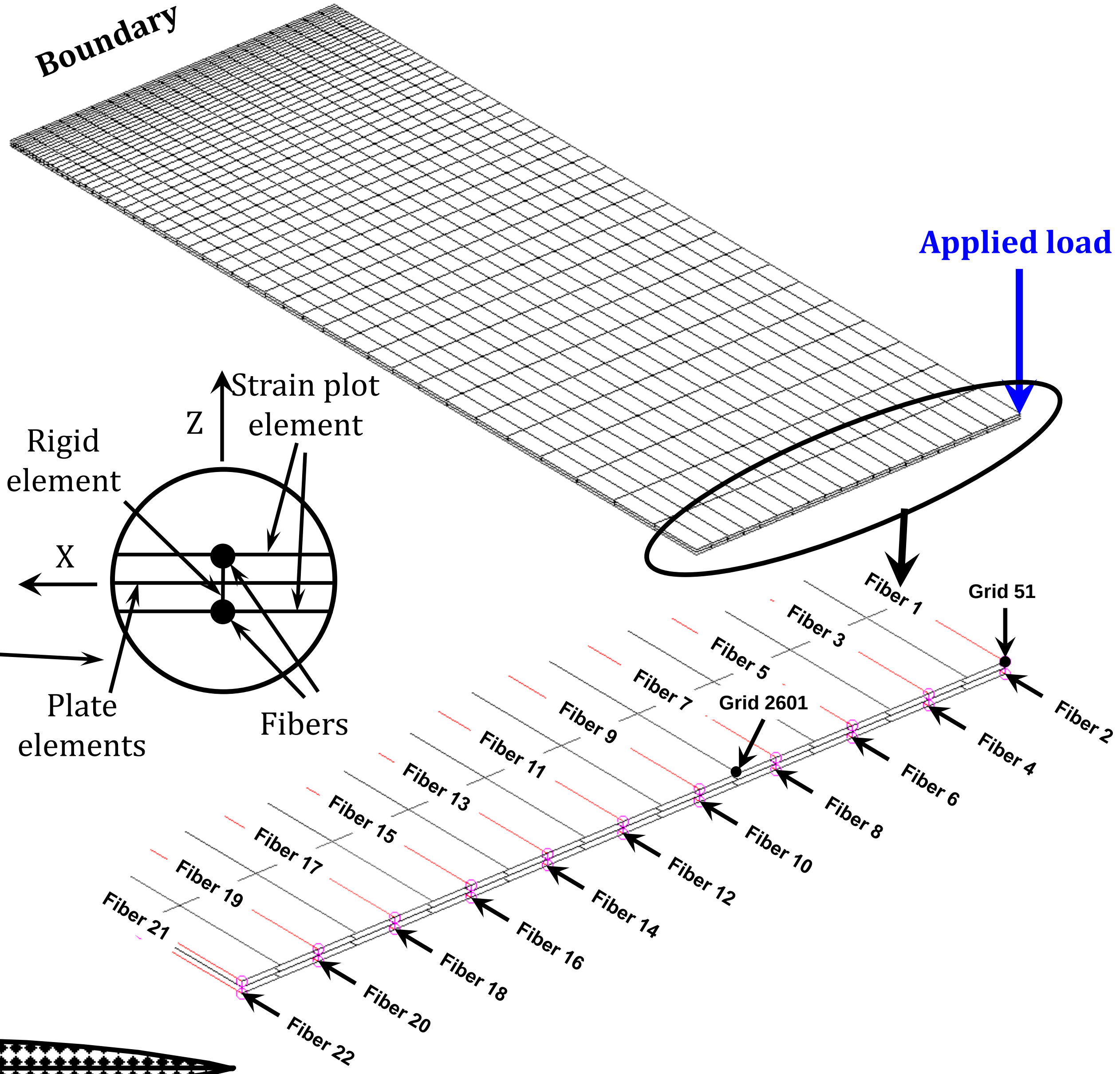
Cantilevered Rectangular Wing Model

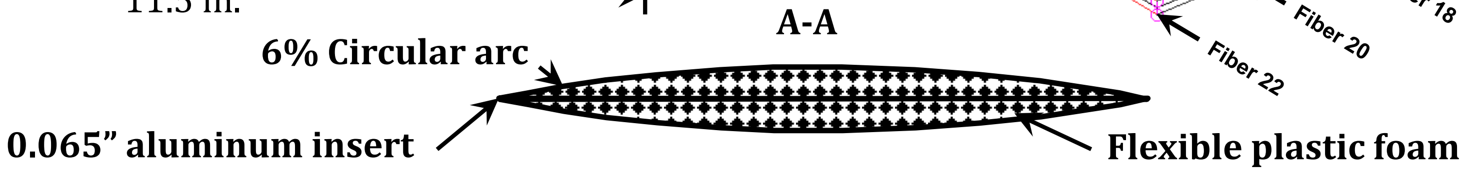
- Configuration of a wind tunnel test article
  - Has aluminum insert (thickness = 0.065 in) covered with 6% circular arc cross-sectional shape (plastic foam)
  - Impulsive load is applied at the leading-edge of wing tip section
- MSC/NASTRAN sol 112: Modal transient response analysis
  - Compute strain
  - Compute deflection & acceleration (**target**)
- Two-step approach
  - Compute deflection and acceleration from computed strain
  - Compare computed deflection and acceleration with respect to target values

<u>22 Simulated FOSS locations</u>

Fiber optic strain sensors: 11(upper) + 11(lower)

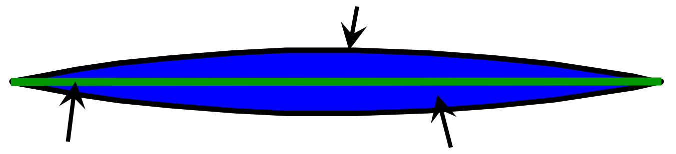






Model Tuning

- Idealization of the plastic foam weight
  - Case 1: equally smeared in aluminum plate.
  - Case 2: lumped mass weight are computed based on 6% circular-arc cross sectional shape.
    - Use structural dynamic model tuning technique
    - Chan-gi Pak and Samson Truong, "Creating a Test-Validated Finite-Element Model of the X-56A Aircraft Structure," *Journal of Aircraft*, (2015), doi: http://arc.aiaa.org/doi/abs/10.2514/1.C033043



Design variables

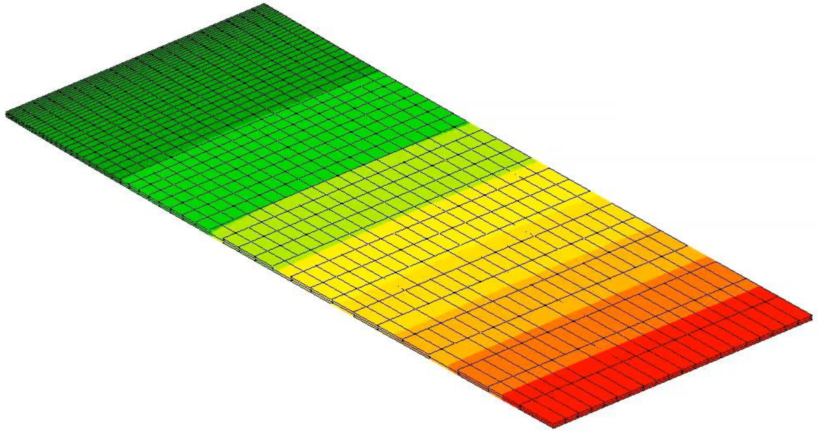
Properties	Case 1 Model	Case 2 Model
E	9847900	**9207766**
G	3639672	**3836570**
density	0.11166	0.1
Foam weight	Smeared	Lumped mass
Total weight	0.3806 lb	0.3806 lb
Xcg	2.28 inch	2.28 inch
Ycg	5.75 inch	5.75 inch
thickness	0.065 inch	0.065 inch

Objective function: frequency error

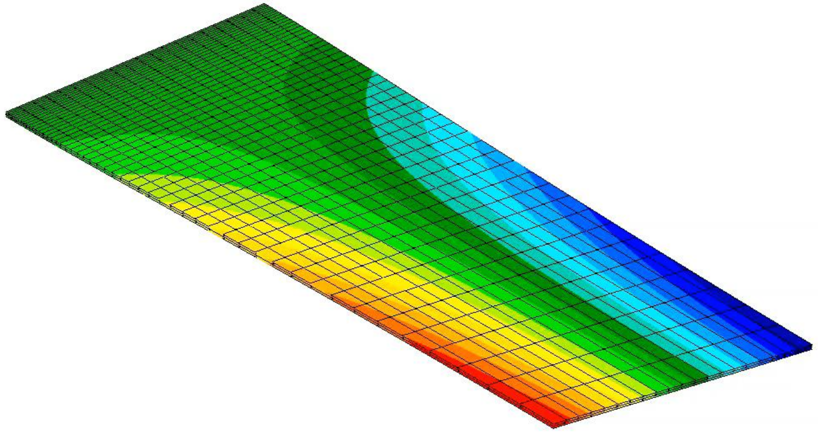
Measured vs. Computed Frequencies

Mode	Measured (Hz)	Case 1 (Hz)	% Error	Case2 (Hz)	% Error
1	14.29	15.09	**5.6**	14.29	**0.0**
2	80.41	77.40	**-3.7**	80.17	**-0.3**
3	89.80	93.57	**4.2**	89.04	**-0.8**
4	N/A	246.37	N/A	248.76	N/A
5	N/A	262.02	N/A	252.41	N/A
6	N/A	455.22	N/A	459.34	N/A
7	N/A	511.27	N/A	485.61	N/A
8	N/A	642.72	N/A	606.65	N/A
9	N/A	722.32	N/A	718.59	N/A
10	N/A	773.93	N/A	747.65	N/A

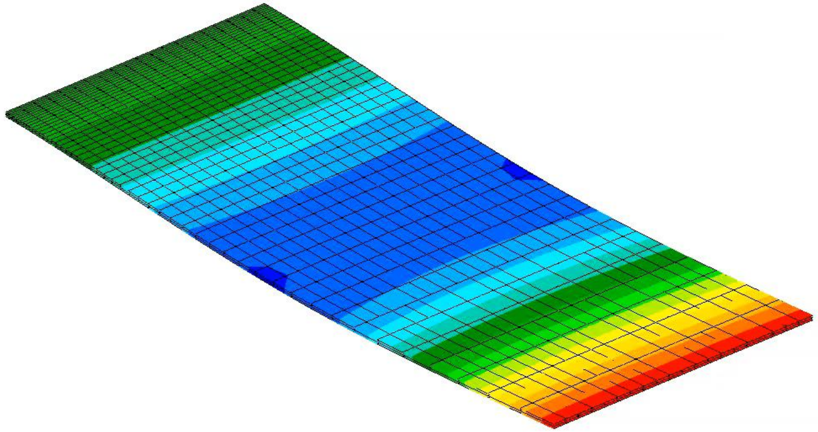
Mode Shapes



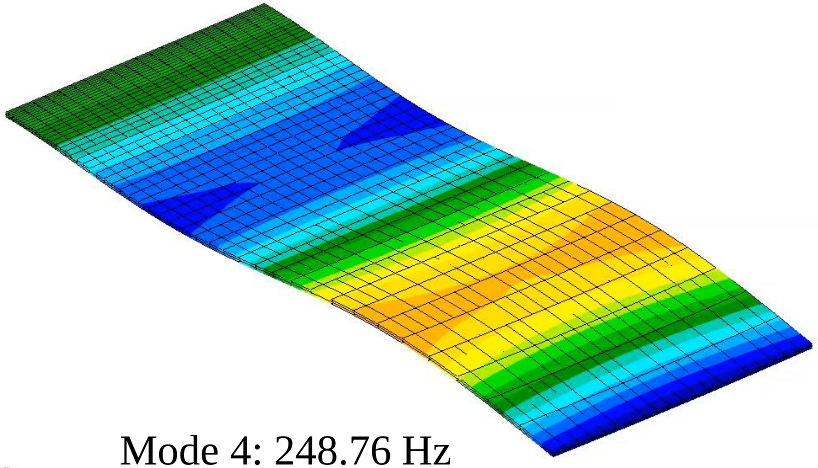
Mode 1: 14.29 Hz



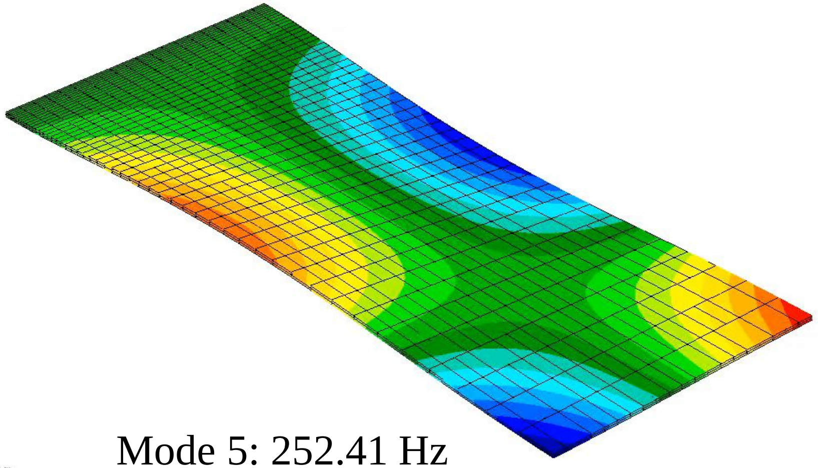
Mode 2: 80.17 Hz



Mode 3: 89.04 Hz



Mode 4: 248.76 Hz



Mode 5: 252.41 Hz

Two Sample Cases

- Case 1 computations
  - Case 1 properties are used to make the target responses.
    - Use NASTRAN modal transient response analysis (sol112)
    - 1200 time steps
  - Mode shapes from Case 1 are used to calculate transformation matrices.
    - Mode shapes are **eigen function**.
  - Frequencies are estimated from strain data computed using Case 1 model.

$$\{\boldsymbol{q}\}_{\boldsymbol{k}} = \begin{bmatrix} \boldsymbol{\Phi}_{\boldsymbol{M}} \\ \boldsymbol{\Phi}_{\boldsymbol{S}} \end{bmatrix} \left([\boldsymbol{\Phi}_{\boldsymbol{M}}]^{\boldsymbol{T}}[\boldsymbol{\Phi}_{\boldsymbol{M}}]\right)^{-1}[\boldsymbol{\Phi}_{\boldsymbol{M}}]^{\boldsymbol{T}}\{\tilde{\boldsymbol{q}}_{\boldsymbol{M}}\}_{\boldsymbol{k}} \qquad \{\ddot{\boldsymbol{q}}\}_{\boldsymbol{k}} = -\begin{bmatrix} \boldsymbol{\Phi}_{\boldsymbol{M}}[\boldsymbol{\omega}_{\boldsymbol{i}}^2] \\ \boldsymbol{\Phi}_{\boldsymbol{S}}[\boldsymbol{\omega}_{\boldsymbol{i}}^2] \end{bmatrix} \left([\boldsymbol{\Phi}_{\boldsymbol{M}}]^{\boldsymbol{T}}[\boldsymbol{\Phi}_{\boldsymbol{M}}]\right)^{-1}[\boldsymbol{\Phi}_{\boldsymbol{M}}]^{\boldsymbol{T}}\{\tilde{\boldsymbol{q}}_{\boldsymbol{M}}\}_{\boldsymbol{k}}$$

From estimation

- Case 2 computations
  - Case 2 properties are used to make the target responses.
    - Use NASTRAN modal transient response analysis (sol112)
    - 1200 time steps
  - Mode shapes from Case 1 are used to calculate transformation matrices.
    - Mode shapes are **comparison function**.
      - ✓ Case 1 model: Not validated model
      - ✓ Case 2 model: Validated model
  - Frequencies are estimated from strain data computed using Case 2 model.

Mode	Measured (Hz)	Case 1 (Hz)	Case2 (Hz)
1	14.29	15.09	14.29
2	80.41	77.40	80.17
3	89.80	93.57	89.04
4	N/A	246.37	248.76
5	N/A	262.02	252.41
6	N/A	455.22	459.34
7	N/A	511.27	485.61
8	N/A	642.72	606.65
9	N/A	722.32	718.59
10	N/A	773.93	747.65

$$\{\dot{\boldsymbol{q}}\}_{\boldsymbol{k}} = \begin{Bmatrix} \dot{\boldsymbol{q}}_{\boldsymbol{M}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{S}} \end{Bmatrix}_{\boldsymbol{k}} = \begin{bmatrix} \boldsymbol{\Phi}_{\boldsymbol{M}} \\ \boldsymbol{\Phi}_{\boldsymbol{S}} \end{bmatrix} \{\dot{\boldsymbol{\eta}}\}_{\boldsymbol{k}} \qquad \{\dot{\boldsymbol{\eta}}\}_{\boldsymbol{k}} = \begin{Bmatrix} \dot{\eta}_1(k) \\ \dot{\eta}_2(k) \\ \vdots \\ \dot{\eta}_i(k) \end{Bmatrix}$$

$$\dot{\eta}_i(k) = \frac{a_{1i}\eta_i(k) + (a_{2i} - 1)\eta_i(k-1)}{2\Delta t}$$

$$\{\boldsymbol{\eta}\}_{\boldsymbol{k}} = \left([\boldsymbol{\Phi}_{\boldsymbol{M}}]^{\mathrm{T}}[\boldsymbol{\Phi}_{\boldsymbol{M}}]\right)^{-1}[\boldsymbol{\Phi}_{\boldsymbol{M}}]^{\mathrm{T}}\{\tilde{\boldsymbol{q}}_{\boldsymbol{M}}\}_{\boldsymbol{k}}$$

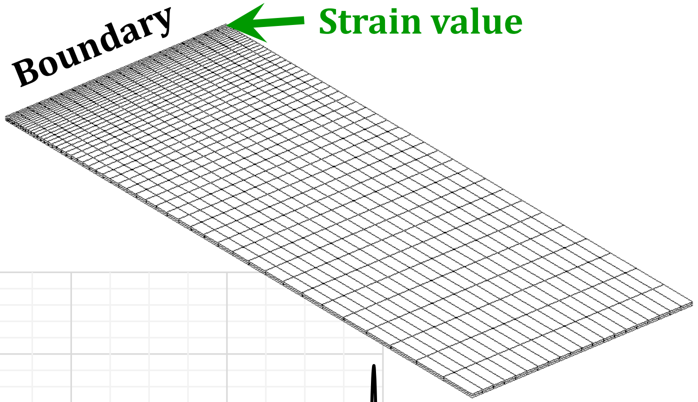
From Case 1 model (comparison function)

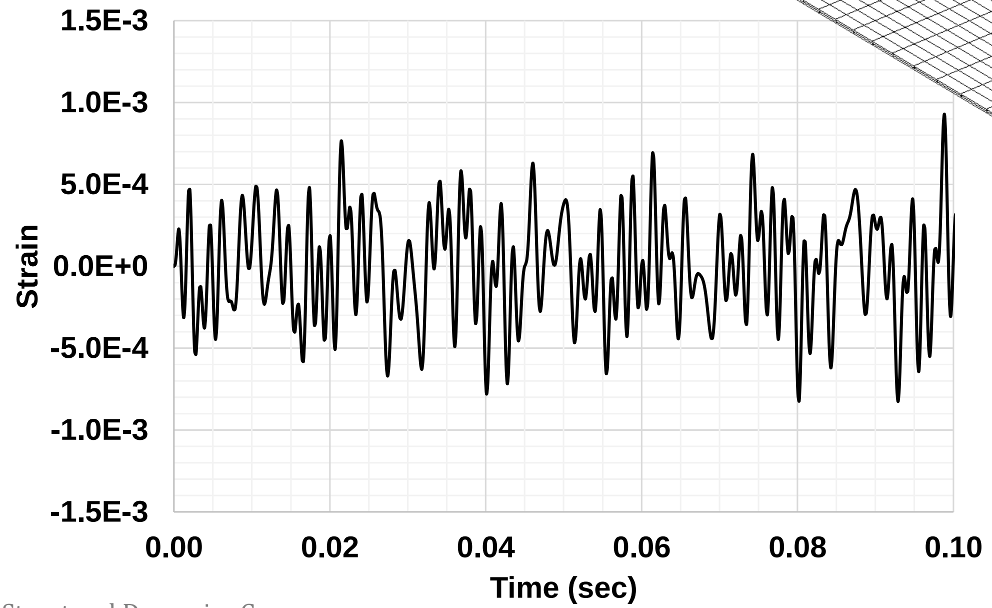
Comparison functions are used for Case 2

Estimated System Frequencies: Case 1

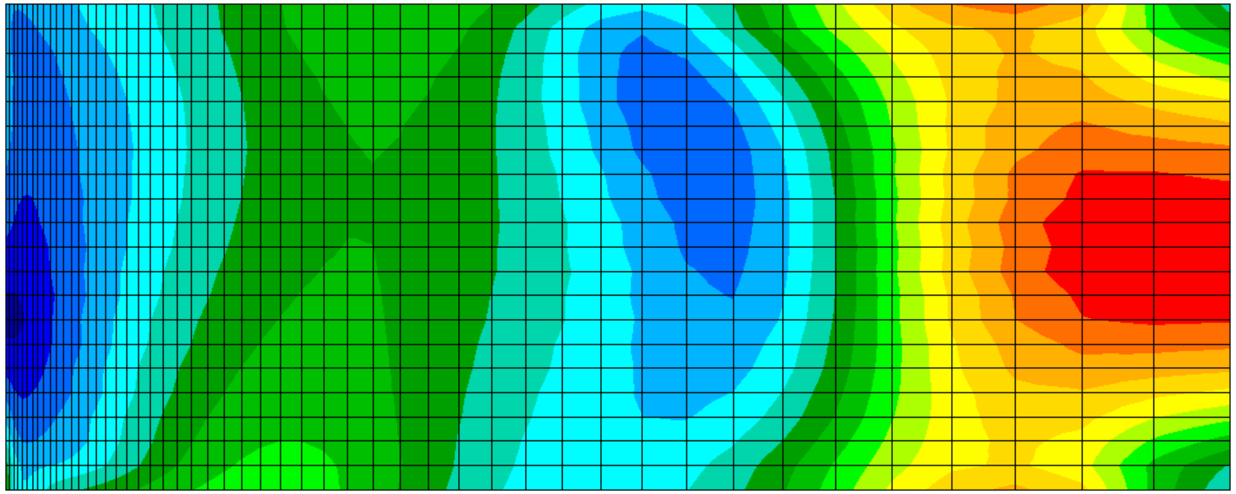
- Use Bierman's U-D Factorization Algorithm
- Number of AR Coefficients = 20
- Covariance matrix resetting interval = 80 time steps
- Forgetting factor = 0.98
- Sampling time = 0.00062667 sec
- Nyquist frequency = 797.9 Hz
- Target frequencies & Time histories of strain: obtained from NASTRAN run
  - Strain values are obtained from the first element of the leading-edge fiber element located at the lower surface.

Mode	Target (Hz)	Estimated (Hz)	% Error
1	15.09	15.09	0.00
2	77.40	77.40	0.00
3	93.57	93.57	0.00
4	246.37	246.37	0.00
5	262.02	262.02	0.00
6	455.22	455.22	0.00
7	511.27	511.27	0.00
8	642.72	642.72	0.00
9	722.32	722.32	0.00
10	773.93	773.93	0.00

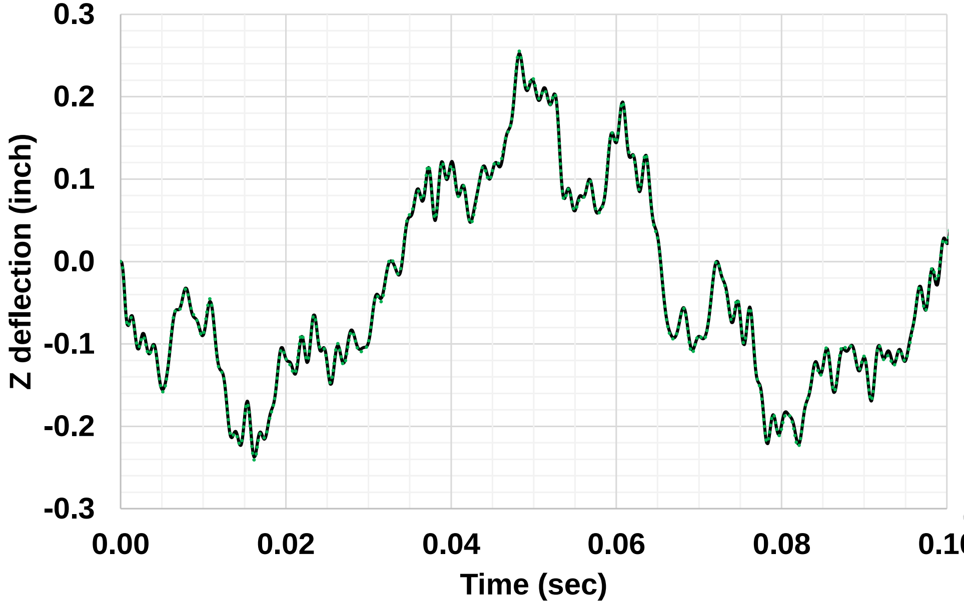




Strain distribution @ T=0.188001 sec

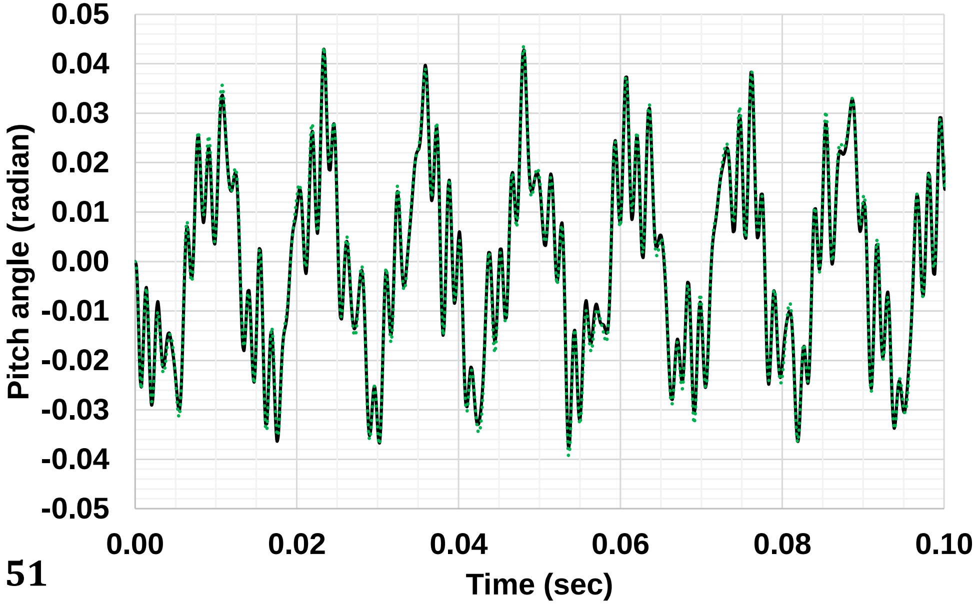


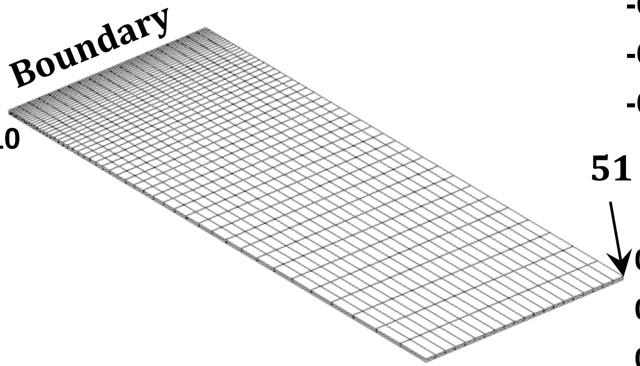
Deflection Time Histories: Case 1

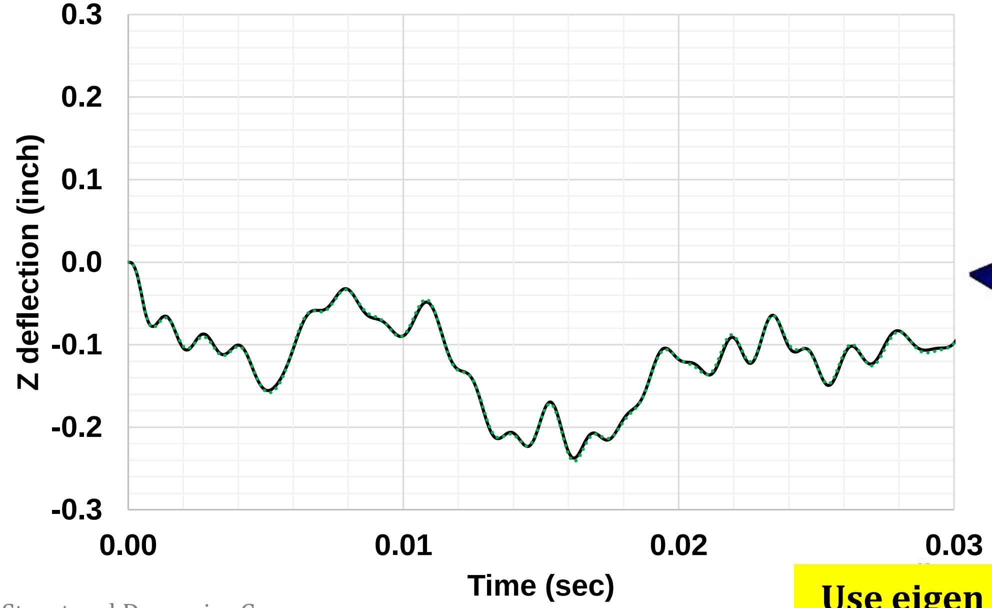


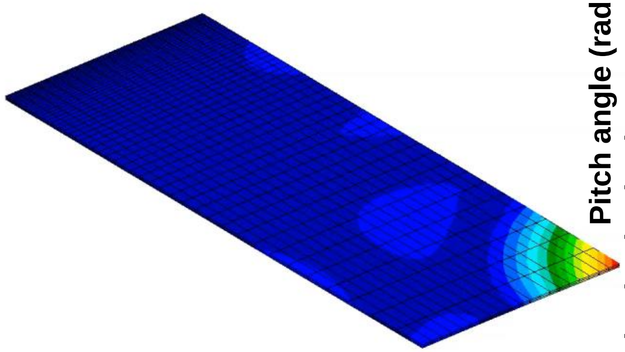
: Target
: Current Method

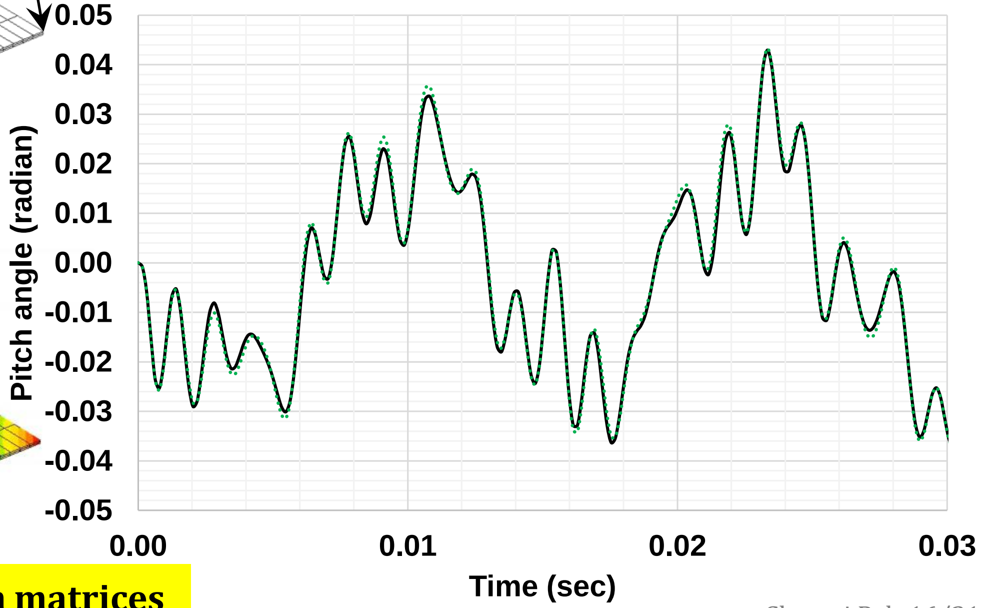
- 22 fibers
- At grid 51









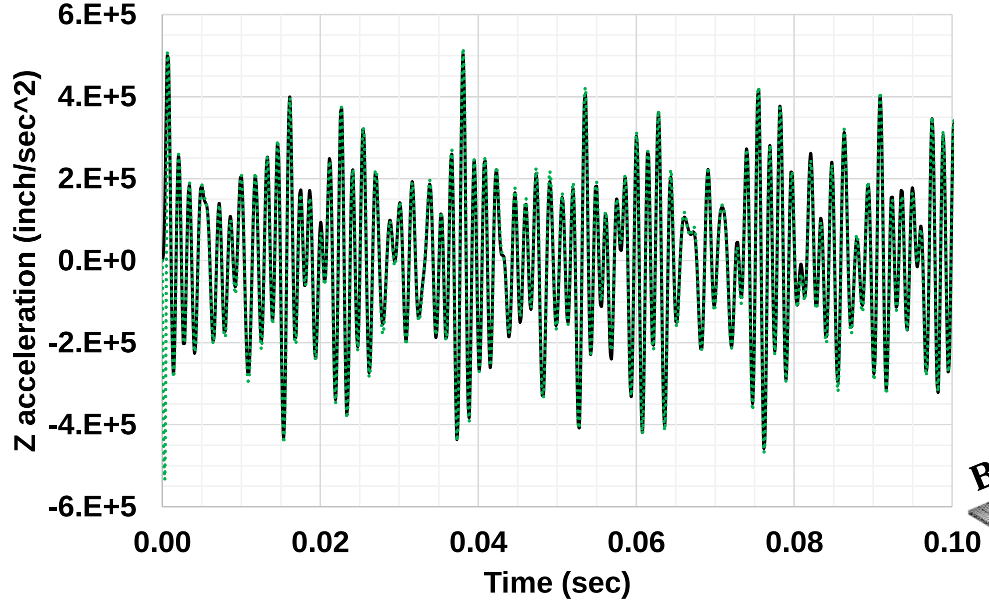


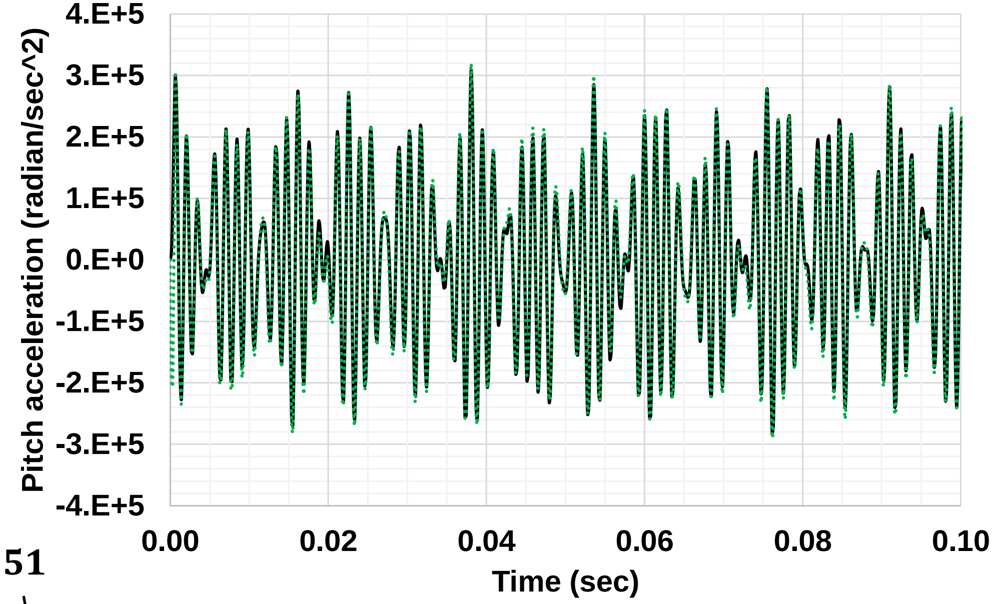
Use eigen functions for the transformation matrices

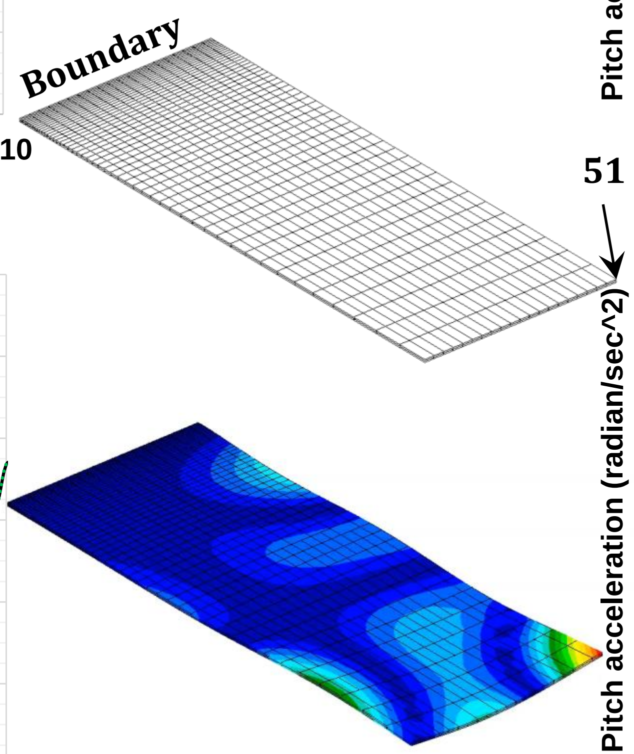
Acceleration Time Histories: Case 1

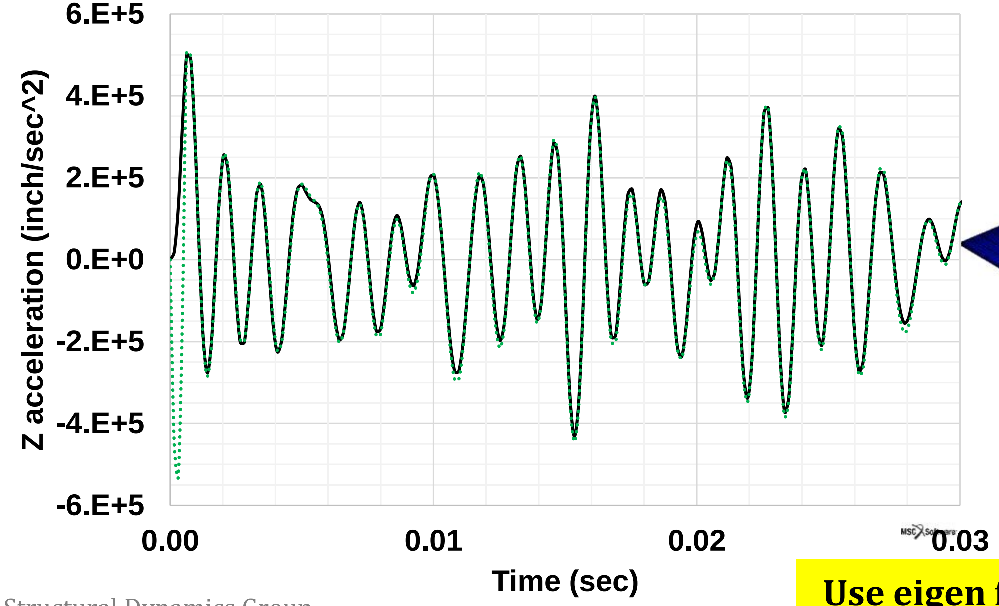
: Target
: Current Method

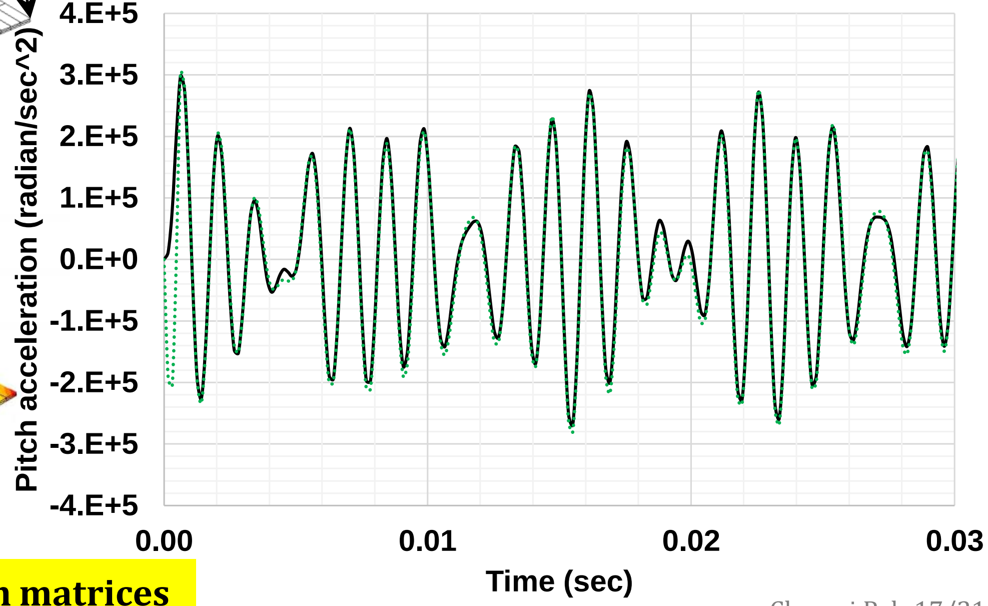
- 22 fibers
- At grid 51





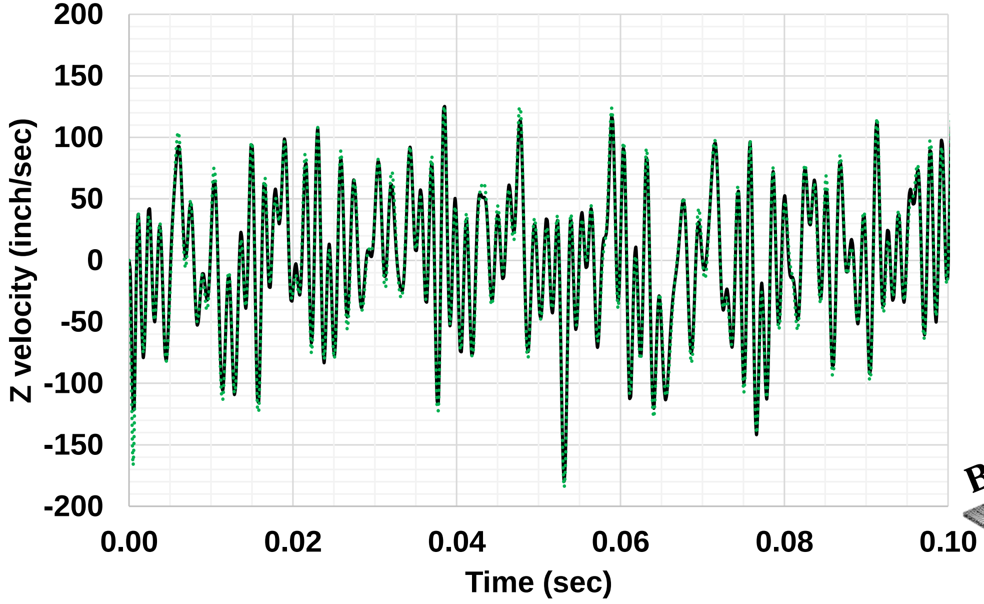






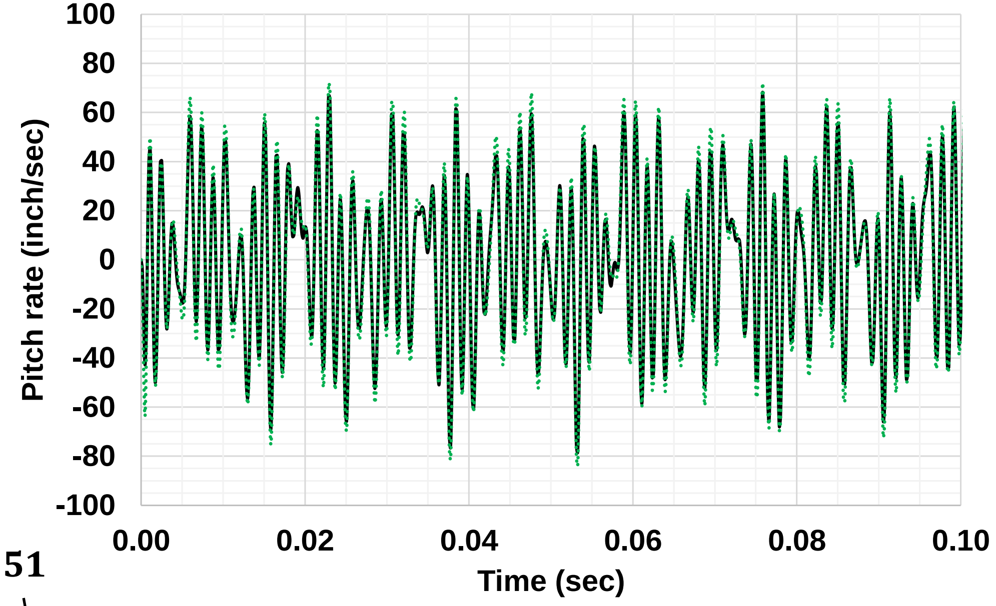
Use eigen functions for the transformation matrices

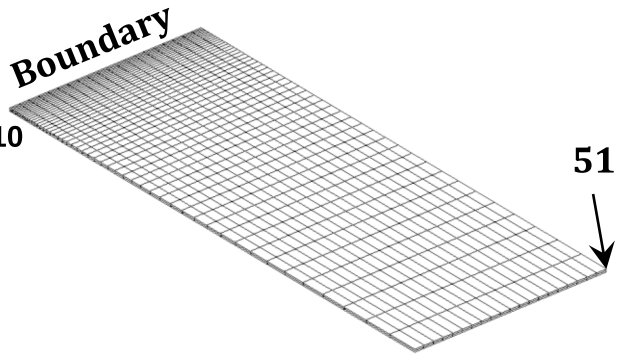
Velocity Time Histories: Case 1

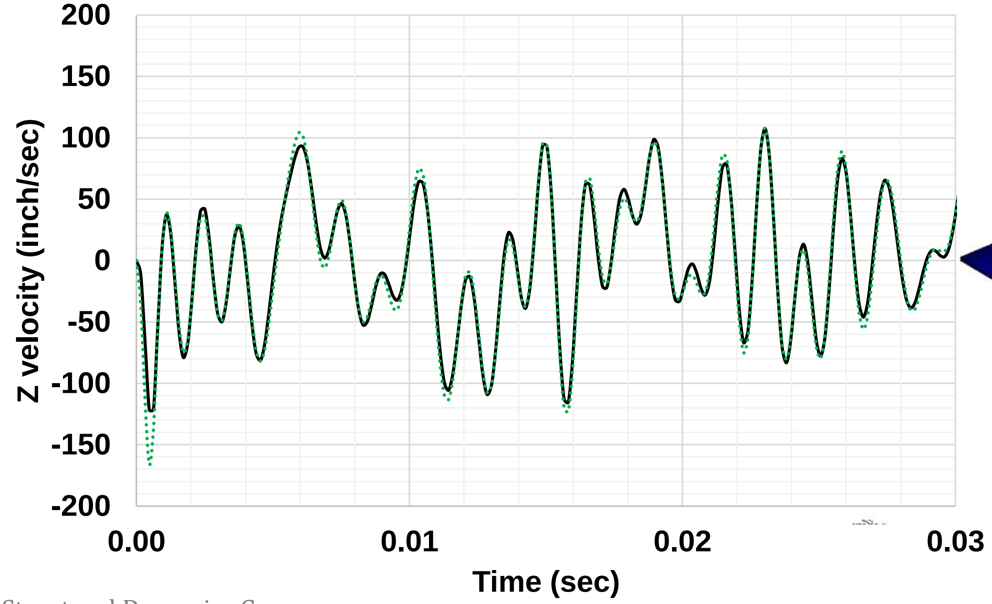


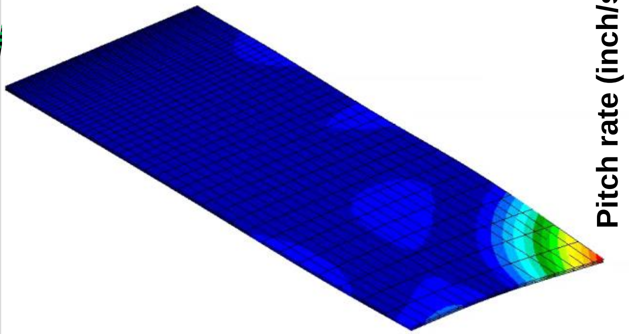
: Target
: Current Method

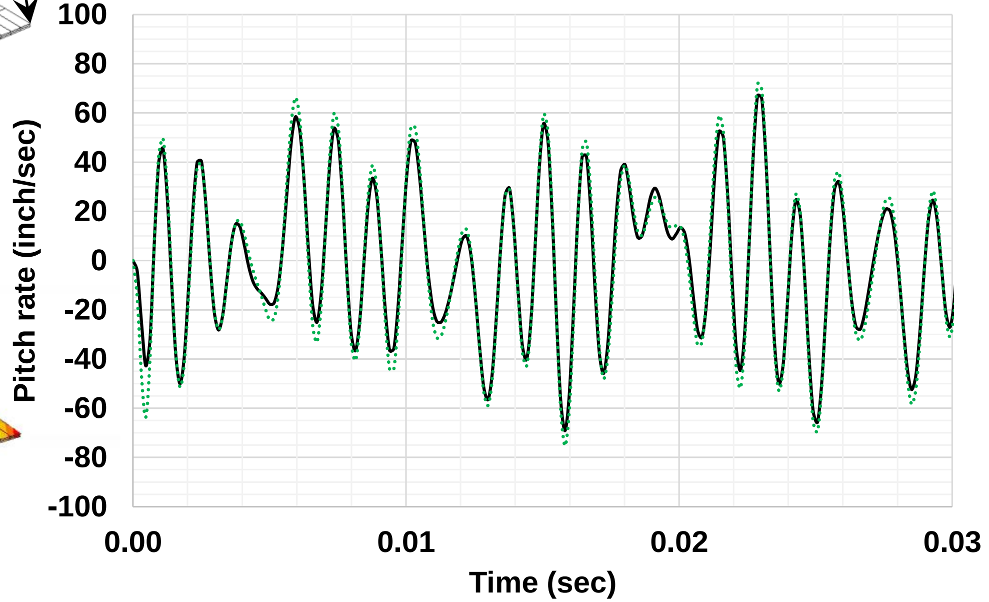
- 22 fibers
- At grid 51





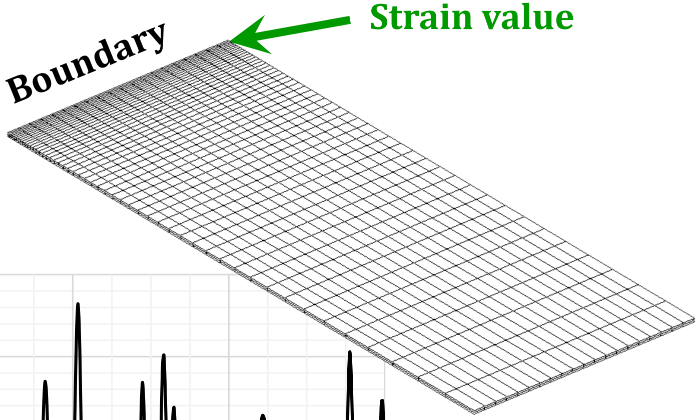


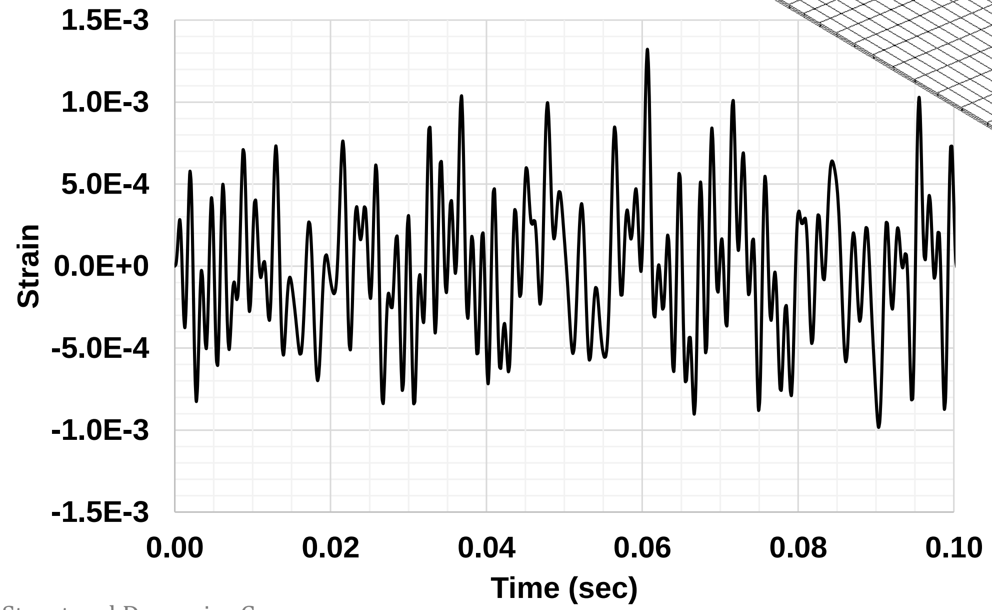




Estimated System Frequencies: Case 2

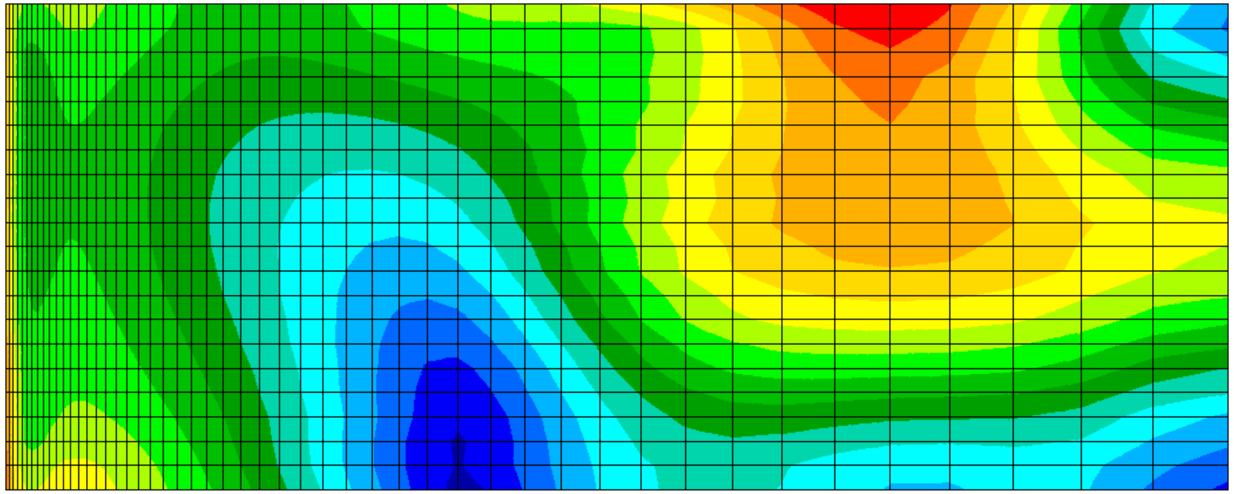
- Use Bierman’s U-D Factorization Algorithm
- Number of AR Coefficients = 20
- Covariance matrix resetting interval = 80 time steps
- Forgetting factor = 0.98
- Sampling time = 0.0006487 sec
- Nyquist frequency = 770.8 Hz
- Target frequencies & Time histories of strain: obtained from NASTRAN run
  - Strain values are obtained from the first element of the leading-edge fiber element located at the lower surface.





Mode	Measured (Hz)	Target (Hz)	Estimated (Hz)	% Error
1	14.29	14.29	14.28	-0.09
2	80.41	80.17	80.18	0.02
3	89.80	89.04	89.05	0.01
4	N/A	248.76	248.77	0.00
5	N/A	252.41	252.41	0.00
6	N/A	459.34	459.34	0.00
7	N/A	485.61	485.61	0.00
8	N/A	606.65	606.65	0.00
9	N/A	718.59	718.60	0.00
10	N/A	747.65	747.66	0.00

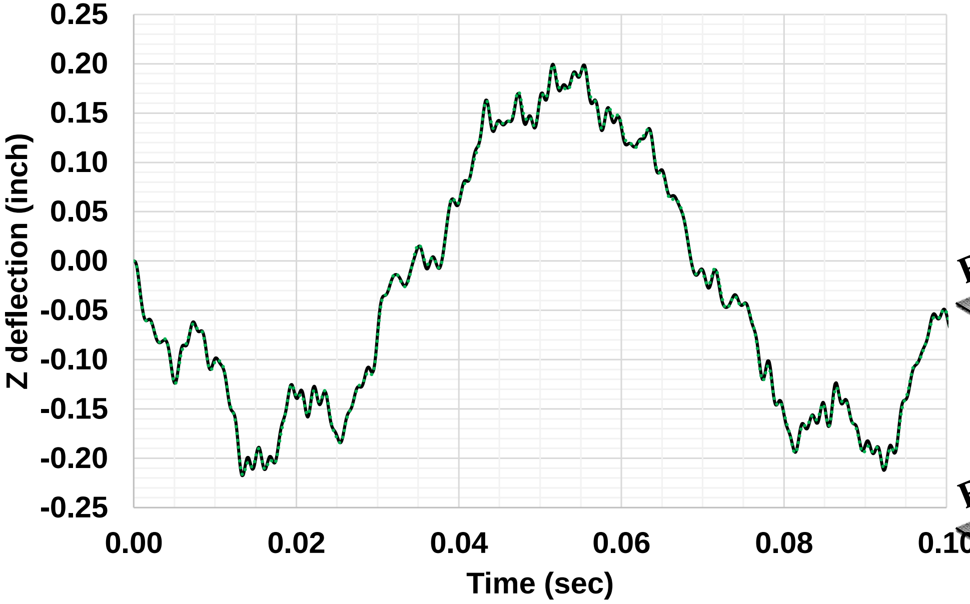
Strain distribution @ T=0.19461 sec

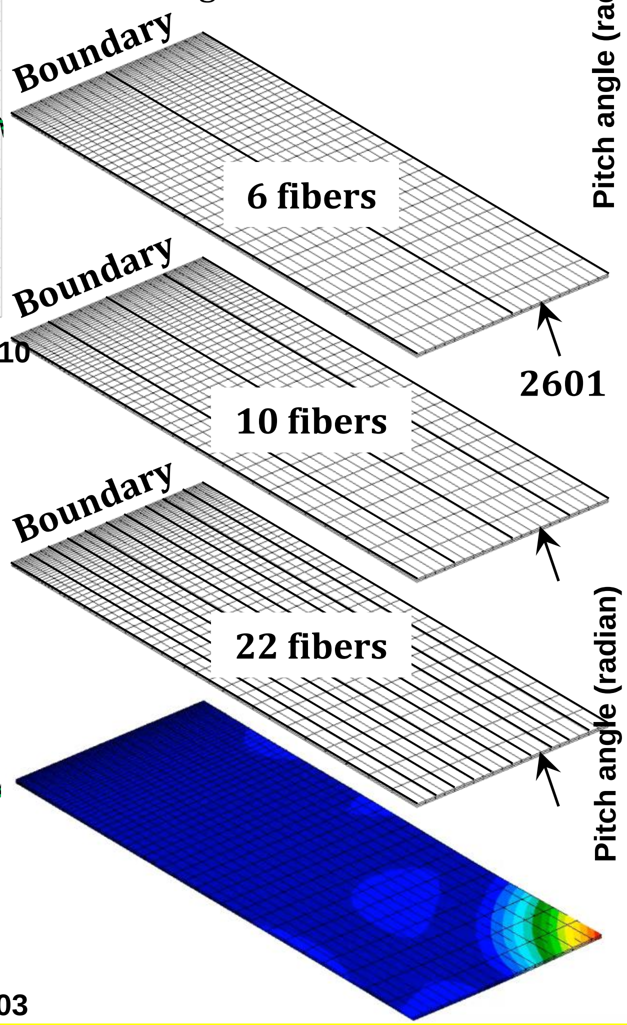


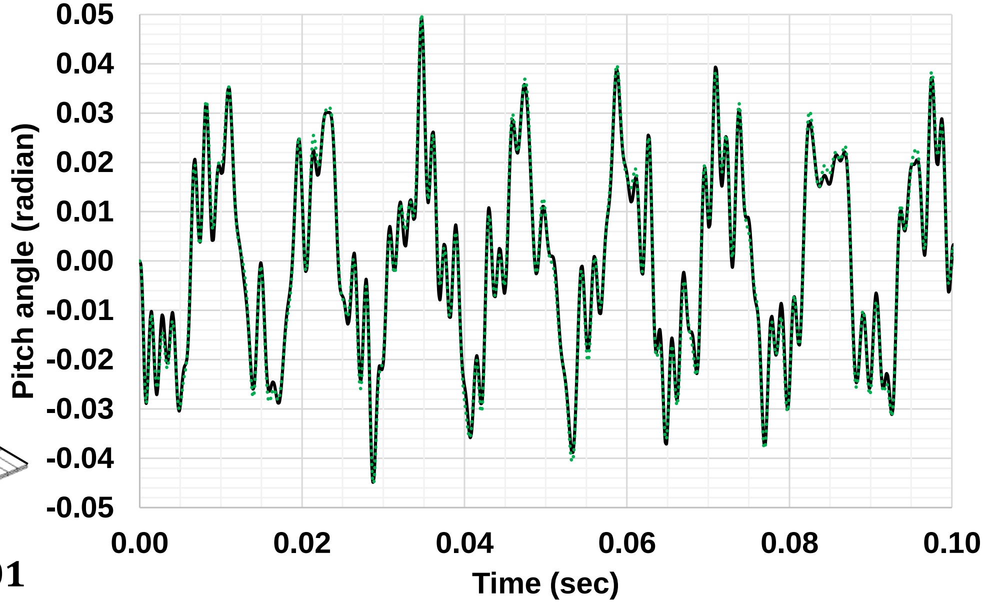
Deflection Time Histories: Case 2

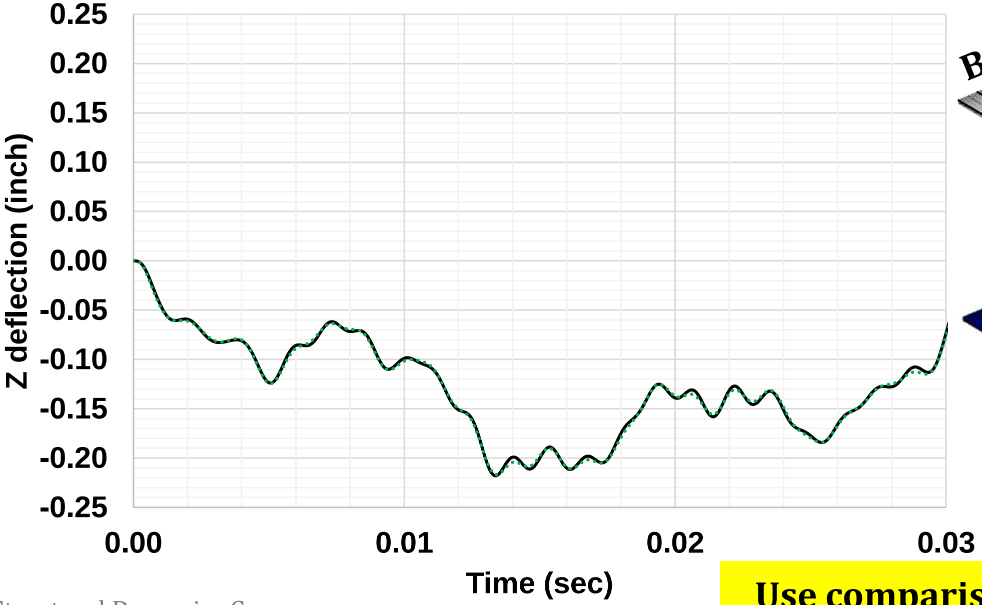
: Target
: Current Method

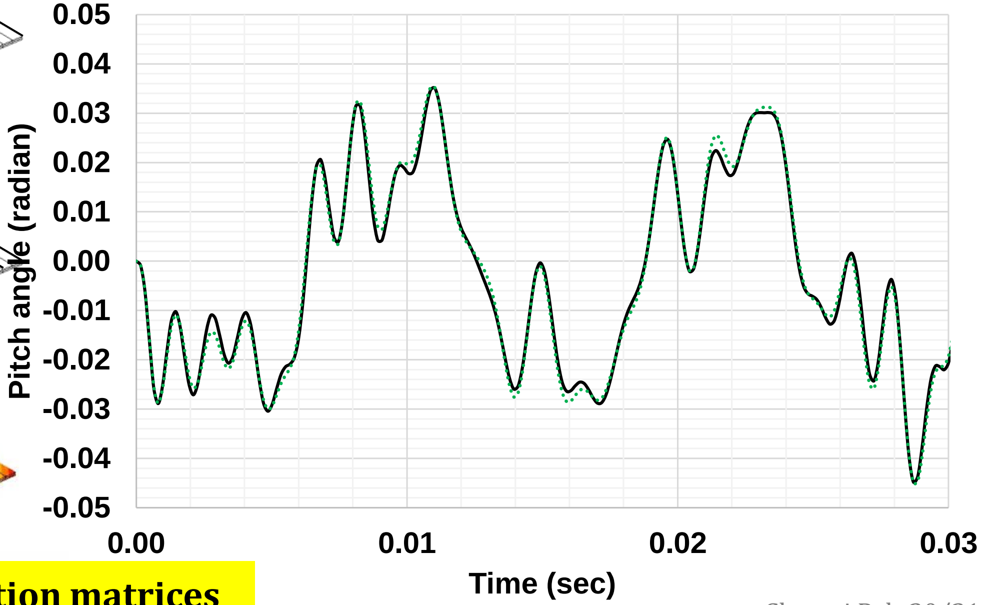
- ❑ 6, 10, & 22 fibers
- ❑ At grid 2601









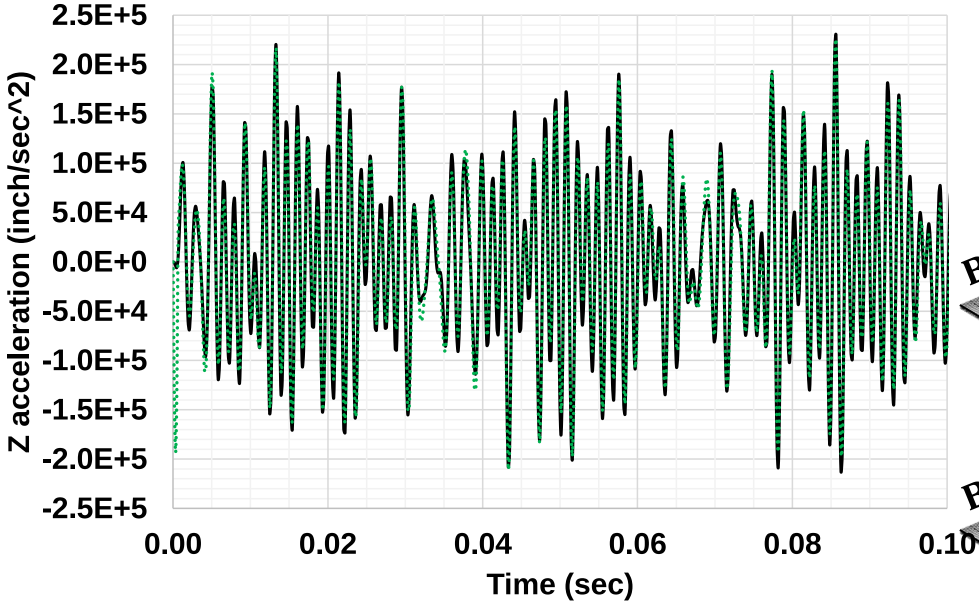


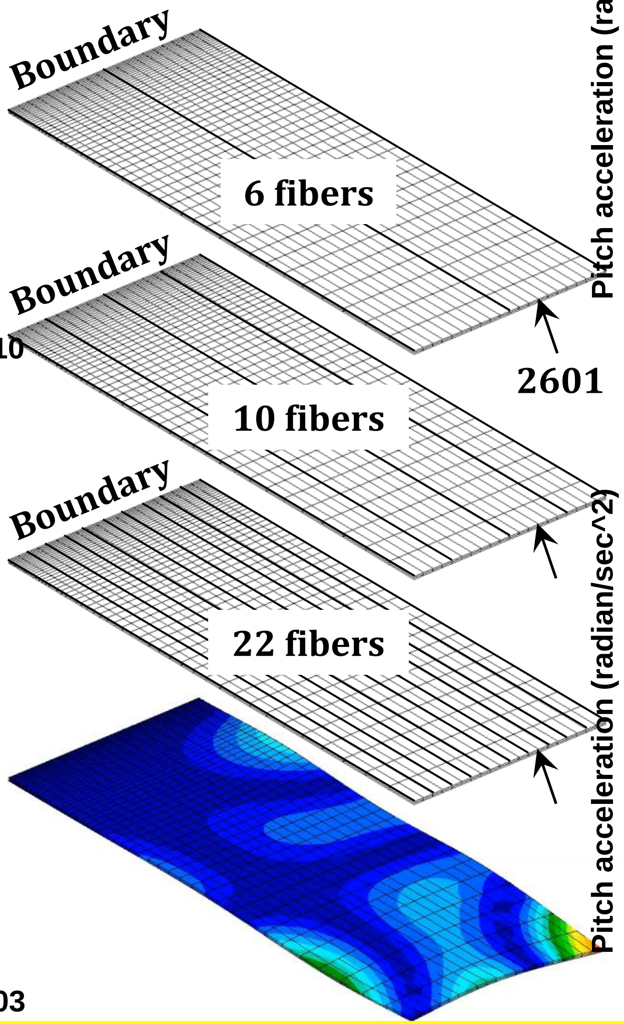
Use comparison functions for the transformation matrices

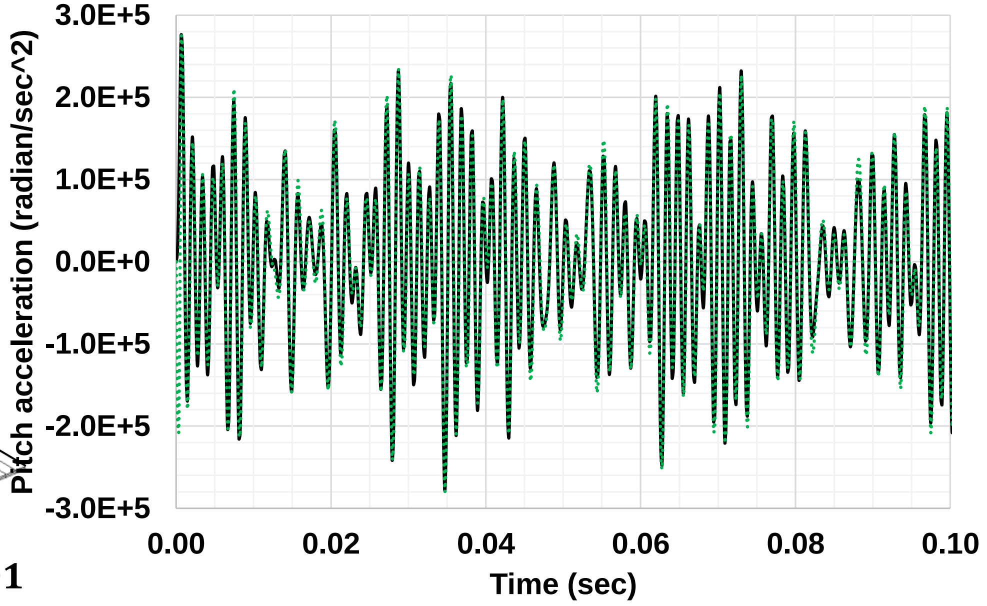
Acceleration Time Histories: Case 2

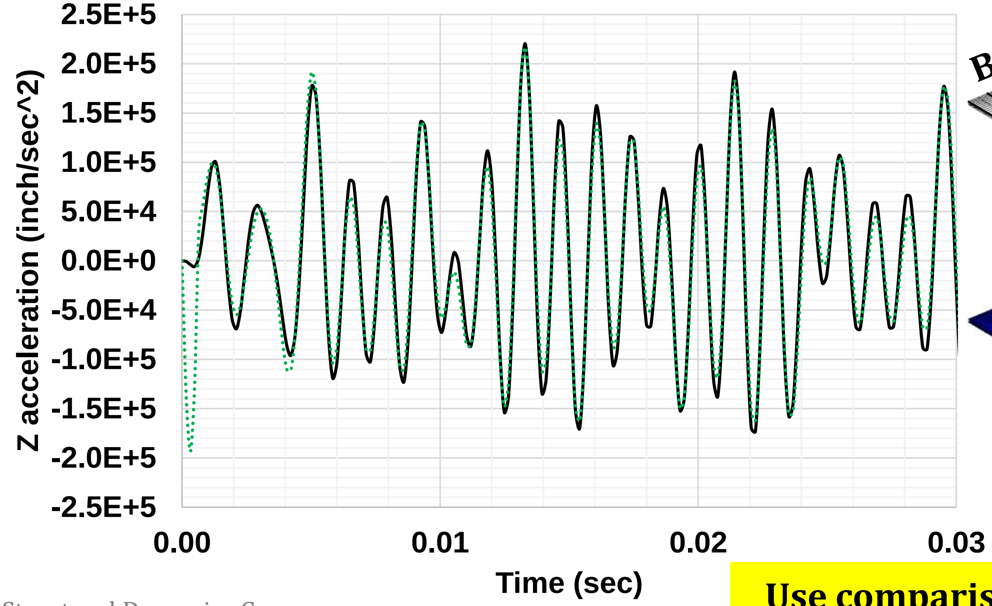
: Target
: Current Method

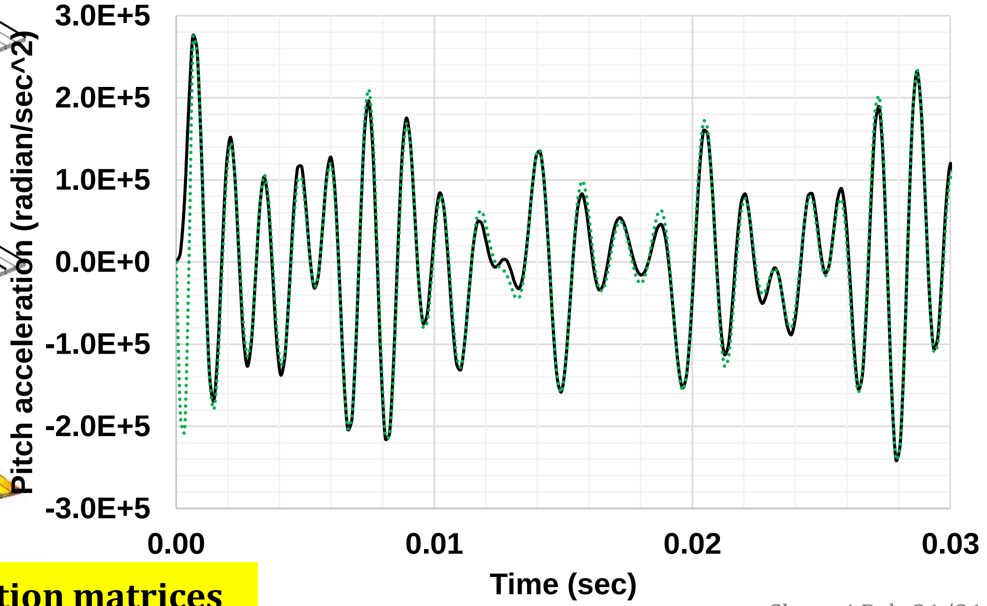
- **6**, 10, & 22 fibers
- At grid 2601





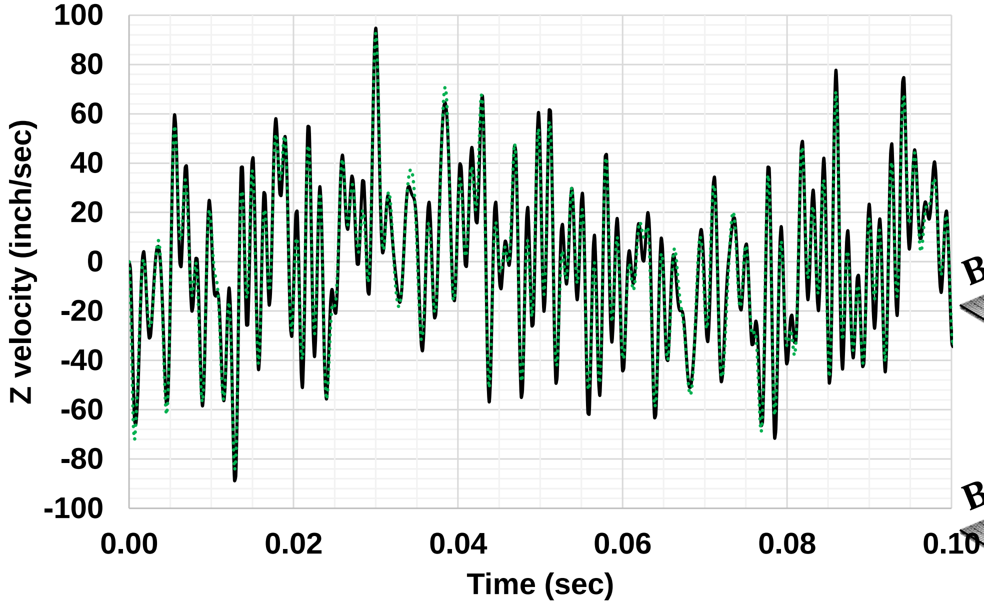






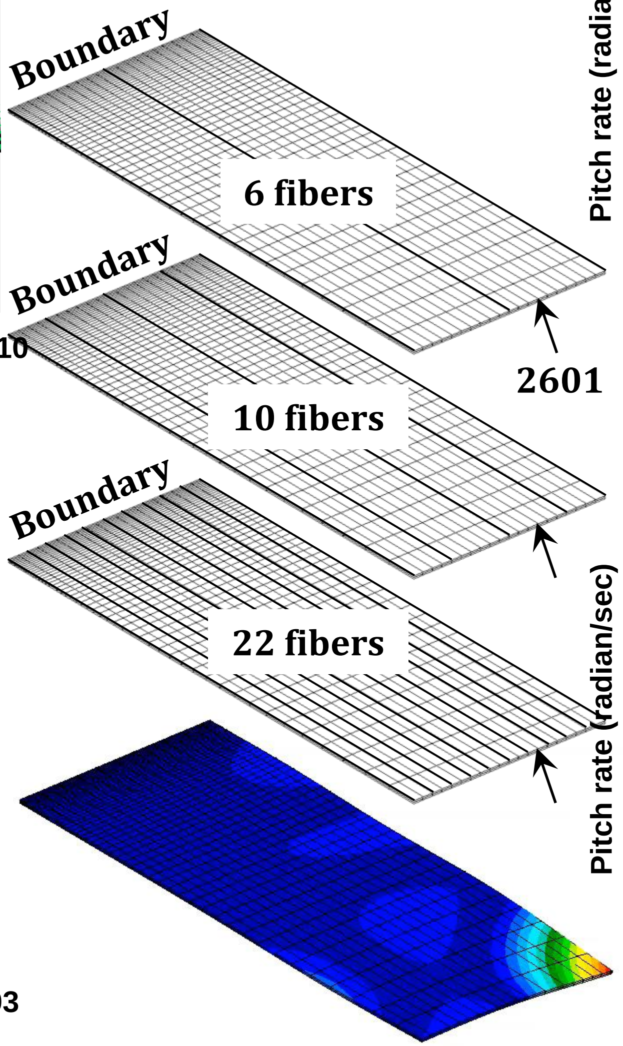
Use comparison functions for the transformation matrices

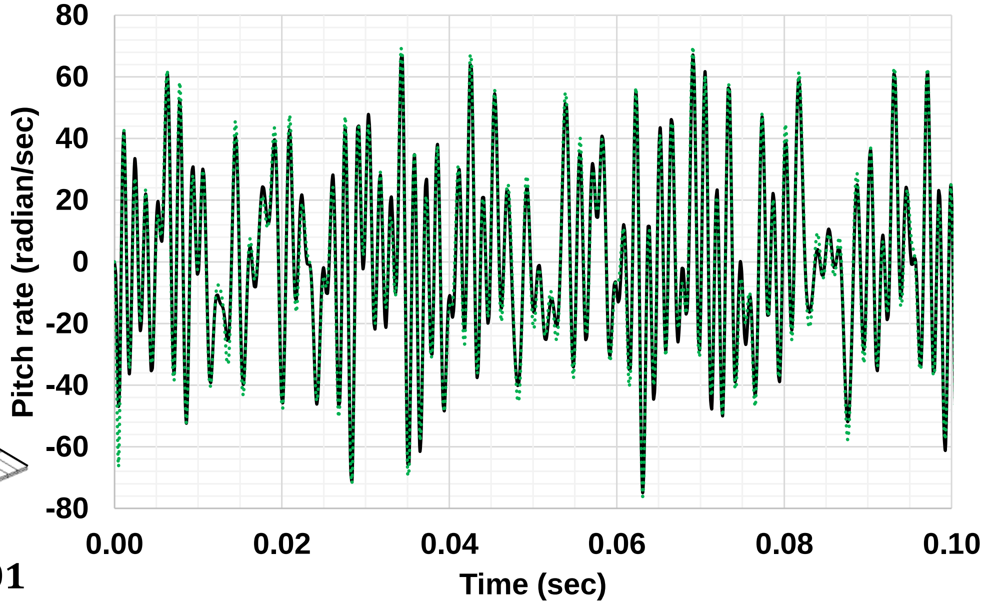
Velocity Time Histories: Case 2

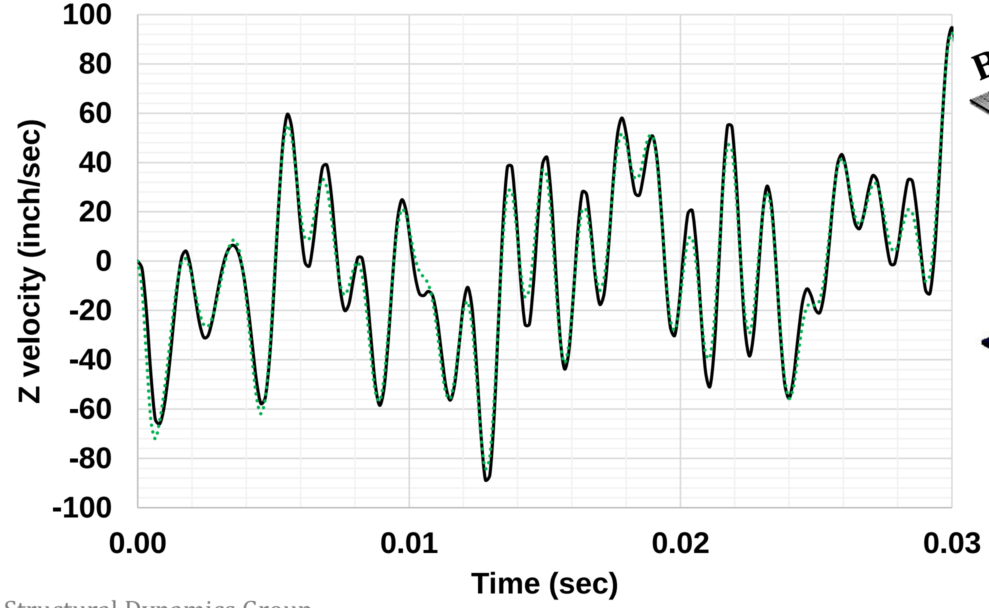


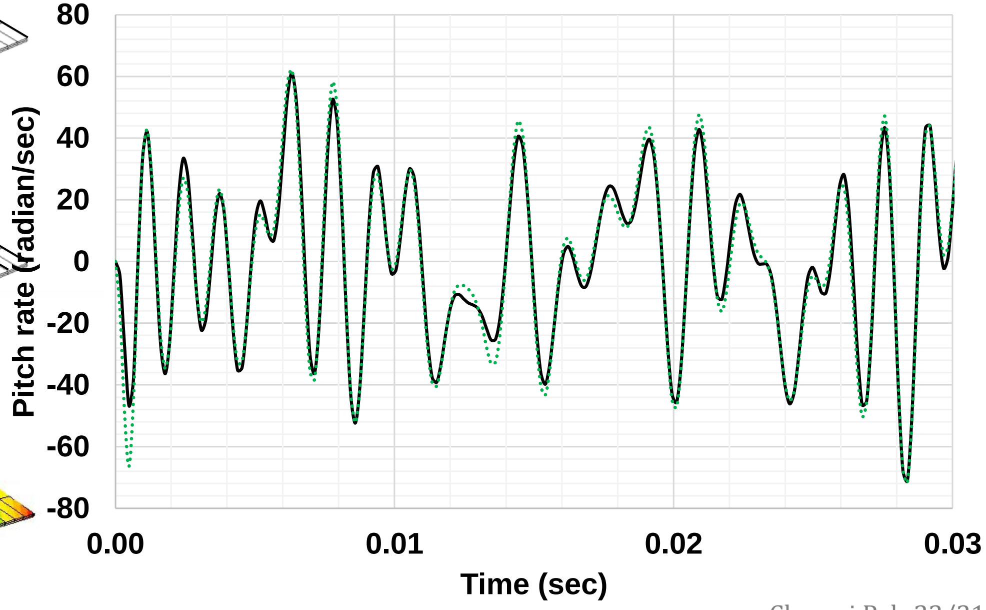
: Target
: Current Method

- **6**, 10, & 22 fibers
- At grid 2601







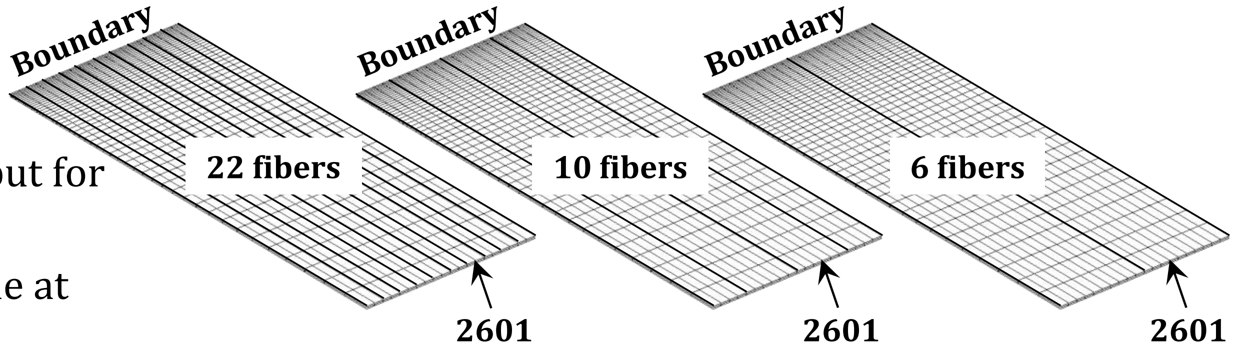


Summary of Computation Error

- % Error $\equiv \frac{\sum_{k=0}^{n}|\text{Current approach }(k)-\text{Target}(k)|}{\sum_{k=0}^{n}|\text{Target}(k)|}$

Model	Grid (# of fiber)	% Error					
		Deflection		Acceleration		Velocity	
		Z	Pitch	Z	Pitch	Z	Pitch
Case 1	51(22)	1.55	5.36	6.42	7.96	10.5	12.0
Case 2	2601(22)	1.38	5.76	16.9	9.84	15.0	11.4
	2601(10)	1.67	5.99	17.0	10.2	15.9	11.7
	2601(6)	1.79	6.35	17.6	10.2	19.0	11.8

- Z deflection errors are the smallest
  - Z deflections are input for the second step.
    - Z deflections along the leading-edge fiber (grid 51) are input for the second step. (master DOF)
    - Pitch angle at grid 51 as well as Z deflection and pitch angle at grid 2601 are output from the second step. (slave DOF) Therefore, it's less accurate than master DOFs.



- Acceleration and velocity errors are bigger than the displacement errors.
- Even six fibers also give good answer.
  - No big difference between 6, 10, & 22 fibers.

Conclusions

- **Acceleration** and **velocity** of the cantilevered rectangular wing is successively obtained using the proposed approach.
  - **Simple harmonic motion** was assumed for the **acceleration** computations.
    - System frequencies are estimated from the time histories of strain measured at the leading-edge of the root section through the use of the **parameter estimation technique** together with the ARMA model.
  - The **central difference** equation with **a linear AR model** is used for the computations of **velocity**.
    - AR coefficients are computed using the estimated system frequencies.
    - Phase shift issue associated with the backward difference equation are overcome with the proposed approach.
  - The total of six fibers provides the good results.
    - Quality of results based on 6, 10, and 22 fibers are **similar**.

Questions ?